#### 6. Graphs & Networks

Visualizing relations

Dr. Thorsten Büring, 29. November 2007, Vorlesung Wintersemester 2007/08





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### Outline

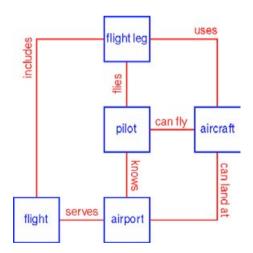
- Graph overview
  - Terminology
  - $\equiv$  Networks and trees
  - Data structures
  - Graph drawing
- $\equiv$  Comparison of graph layouts
- $\equiv$  Graph visualization examples
  - Social networks
  - Copurchase network
  - Music network
  - Transportation network
- $\equiv$  Case study: Telephone network visualizations
- $\equiv$  Comparing node-link and matrix representations
- $\equiv$  Interaction and animation



## **Graph Overview**

- $\equiv$  Graph definition: an abstract structure that is used to model information
- Can represent any information that can be modeled as objects and connections between those objects
- $\blacksquare$  Objects represented by vertices
- $\equiv$  Relations between objects represented by edges
- E Commonly visualized as node-link diagrams
- Example domains
  - World Wide Web
  - Telephone networks
  - Financial transactions
  - Call graph in software engineering (which functions call which other functions)
  - CVS repositories
  - Social networks
  - Transportation networks
  - Co-citations...
- $\equiv$  Graphs in InfoVis shall facilitate the understanding of complex patterns





Automatically generated airline database schema, Tamassia et al. 1988



# Challenges in Graph Drawing

- $\equiv$  Graph Visualization (layout and positioning)
  - $\equiv$  How to present a graph to convey the most information and to make it easy to read and interpret it

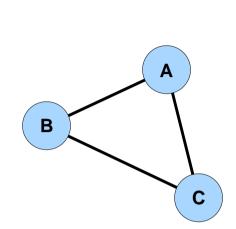
 $\equiv$  Scale

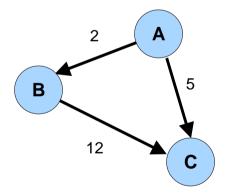
- $\equiv$  Performance of layout algorithms
- $\equiv$  Limited real estate of display area
- $\equiv$  Navigation and Interaction
  - $\equiv$  How to enable the user to move around the graph and inspect portions of the graph in detail



# **Graphs Terminology**

- $\equiv$  Graph consists of
  - $\equiv$  Nonempty set of vertices (points)
  - $\equiv$  Set of edges that link together the vertices
- Undirected graph
- $\equiv$  Directed graph (usually indicted by arrows)
- $\equiv$  Mixed graph contains both directed and undirected graphs
- $\equiv$  Unweighted vs. weighted (nominal, ordinal quantitative) edges
- $\equiv$  Degree of a vertex: the number of edges connected to it
- $\equiv$  In-degree and out-degree for directed graphs
- Adjacency
  - $\equiv$  Two edges sharing a common vertex
  - $\equiv$  Two vertices sharing a common edge



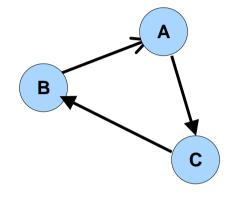




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# Graphs Terminology

- Path: a traversal of consecutive vertices along a sequence of edges
- $\equiv$  Length of the path: number of edges that are traversed along the path
- $\equiv$  Simple path: no repeated vertices within the path
- $\equiv$  Cycle: a path in which the initial vertex of the path is also the terminal vertex of the path
- Acyclic: a simple directed graph not containing any cycles



Directed Graph Cycle



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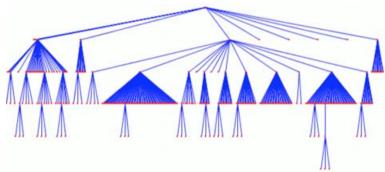
# **Special Types of Graphs**

#### Network

- Directed Graph
- $\equiv$  Usually weighted edges
- $\equiv$  No topological restrictions
- Examples: social, economic, transportation networks

Tree

- $\equiv$  No cycles
- $\equiv$  Usually directed edges
- $\equiv$  Usually special designated root vertex
- $\equiv$  Example: organizational chart
- $\equiv$  Will be topic of next lecture!



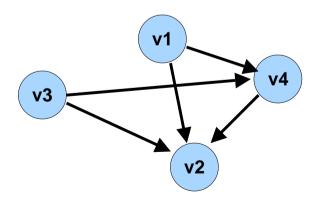
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### **Data Structures for Graphs**

- $\equiv$  Storing and processing a graph on a computer
- $\equiv$  Adjacency List usually used for graphs with small numbers of edges
- Adjacency Matrix allows powerful matrix operations but is often more memory demanding
  - $\equiv$  Row: edges leaving the vertex
  - $\equiv$  Column: edges entering the vertex
- $\equiv$  Example for directed graph



v1 -> v2 -> v4	
v2 ->	
v3 -> v2 -> v4 v4 -> v2	
v4 -> v2	

	v1	v2	v3	v4
v1	0	1	0	1
v2	0	0	0	0
v3	0	1	0	1
v4	0	1	0	0



# Graph Drawing

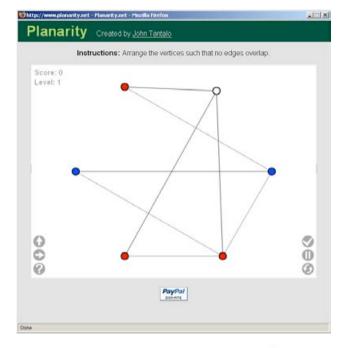
- $\equiv$  Many ways to draw a graph
- $\equiv$  Vertices are usually represented by circles
- $\equiv$  Edges are usually represented by open curves between vertices
- Node-link diagram
- $\equiv$  Potential encoding attributes
  - Color
  - ∃ Size
  - $\equiv$  Form / Shape
- $\equiv$  Labeling is often difficult due to clutter

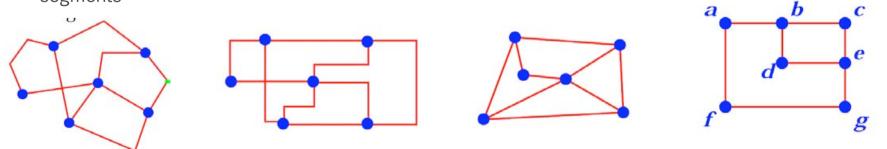


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## **Graph Drawing**

- $\equiv$  Layout algorithms can be categorized by the type of layout they generate
- $\equiv$  Planar: edges do not intersect
- Straight, polyline (edge with bends) or curved lines
- Orthogonal: polyline drawing that maps each edge into a chain of horizontal and vertical segments



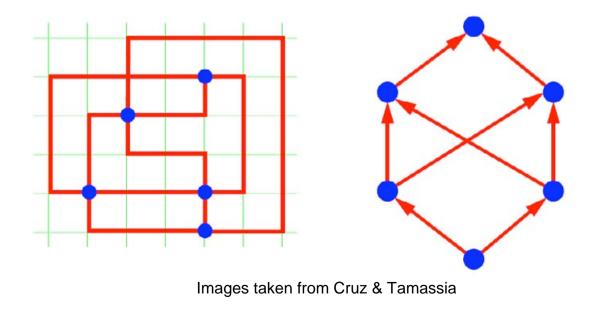


Images taken from Cruz & Tamassia



# Graph Drawing

- Grid-based: vertices (and bends of the edges) have integer coordinates implies minimum distance between vertices and nonincident edges
- Upward / downward drawing for directed acyclic graphs: make edges flow in the same direction, e.g. for visualizing hierarchies





# Layout Aesthetics

- Minimize crossing keep the number of times that lines cross to a minimum (hardly applicable in interactive systems)
- $\equiv$  Minimize area keep the area that the graph takes up to a minimum by producing a compact graph
- $\equiv$  Minimize the sum of the edge lengths
- $\equiv$  Obtain a uniform edge length try to keep each of the edges at the same lengths
- $\equiv$  Minimize bends keep the number of times there is a bend to a minimum
- $\equiv$  Display symmetry of graph structure
- $\equiv$  Maximize minimum angles between edges
- **—** ...



## **Empirical Results**

#### ■ Purchase 1997

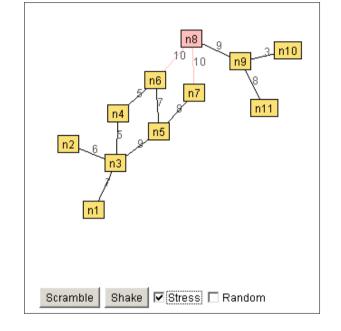
- $\equiv$  Compare task performance on five pairs of graphs
- Graph pairs differed according to numbers of edge bends, edge crosses, maximizing the minimum angle, orthogonality and symmetry
- $\equiv$  Result: Reducing crossings is by far most important
- $\equiv$  Ware et al. 2002
  - $\equiv$  Experimental task: finding the shortest path in spring layout graphs
  - $\equiv$  Results indicate the following prioritization of metrics
    - $\equiv$  Geometric length of the path (implicit property of a graph)
    - $\equiv$  Continuity (keeping multi-edge paths as straight as possible)
    - $\equiv$  Number of edge-crossings



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#### Spring Embedder

- Force-directed model for graph layout
- Eades 1984
- $\equiv$  Intuitive approach: apply physical model of forces
  - $\equiv$  Every vertex is considered a steel ring
  - $\equiv$  Every edge a spring
- Resulting layout represents a configuration of minimum energy (force exerted on each ring is 0)
- $\equiv$  Can produce well-balanced, symmetrical graphs
- Problem: time consuming quality of the graph depends on the number of full iterations (visit all pairs of vertices to calculate the effect of the forces) demo
- Overview of graph drawing algorithms: Pajntar 2006 (http://kt.ijs.si/dunja/SiKDD2006/Papers/Pajntar.pdf)
- Graph drawing library AGD: http://www.ads.tuwien.ac.at/AGD/
- Graph drawing tutorial: http://www.cs.brown.edu/~rt/papers/gdtutorial/gd-constraints.pdf



Spring embedder Java applet + source code http://www.inf.uni-konstanz.de/algo/lehre/ss04/gd/demo.html



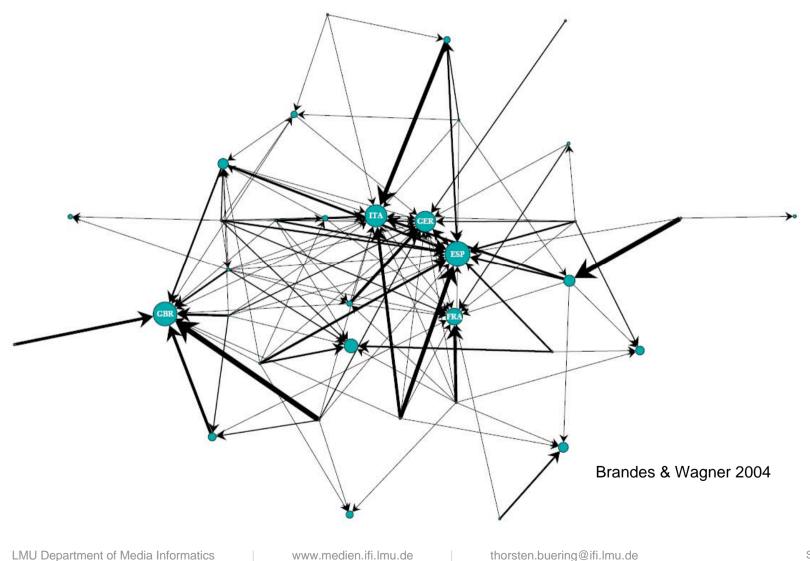
Outline

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  - ∃ Terminology
  - $\equiv$  Networks and trees
  - Data structures
  - **∃** Graph drawing
- $\equiv$  Comparison of graph layouts
- $\equiv$  Graph visualization examples
  - Social networks
  - $\equiv$  Copurchase network
  - Music network
  - ∃ Transportation network
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- $\equiv$  Interaction and animation



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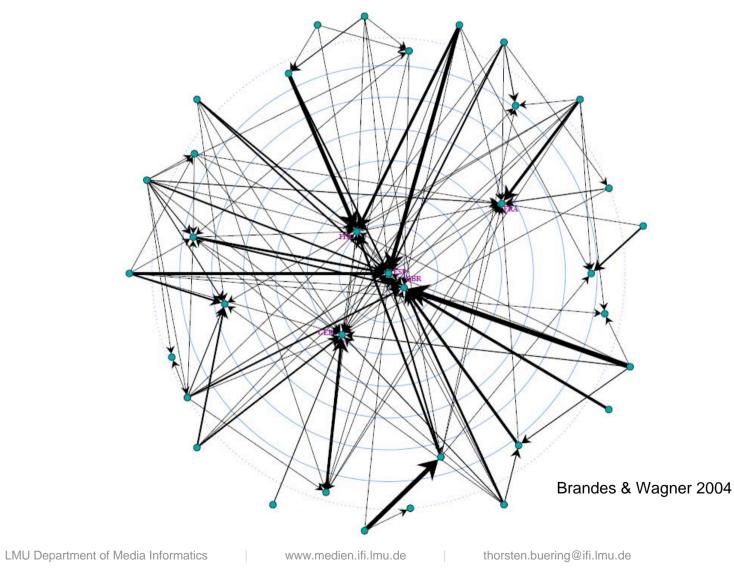
## **Graph Layout Example**

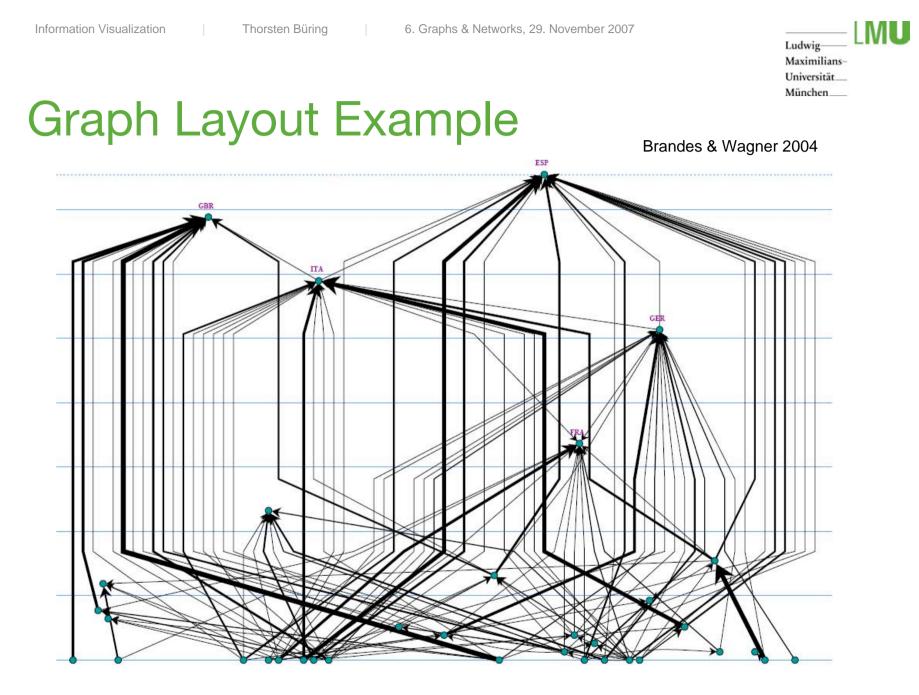




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## **Graph Layout Example**





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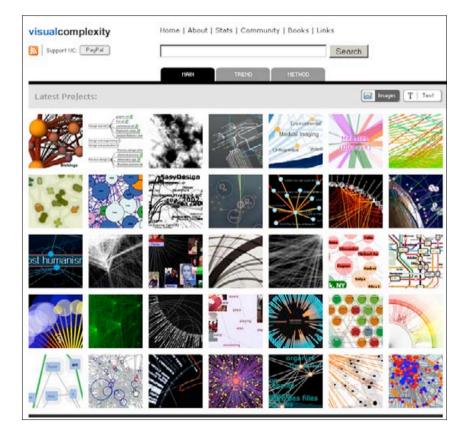
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Various Examples of Graph Drawings

■ http://www.visualcomplexity.com/

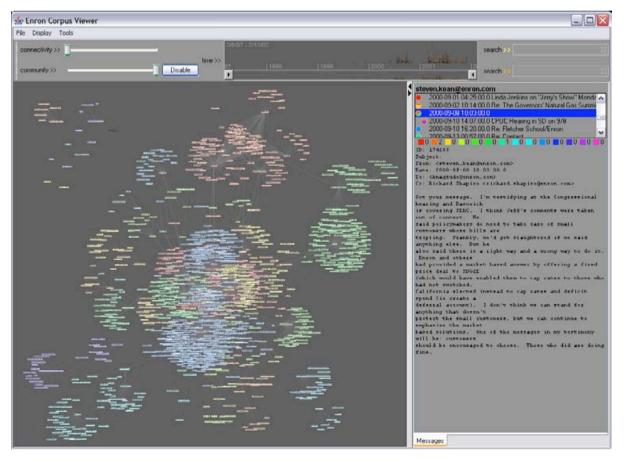




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## **Social Network**

#### Exploring Enron: http://jheer.org/enron/



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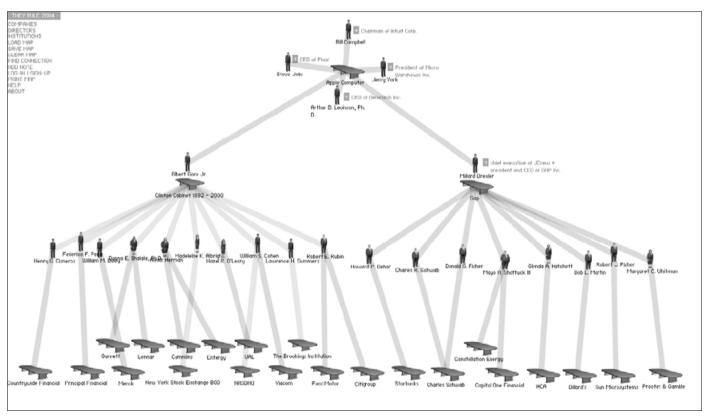
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#### **Social Network**

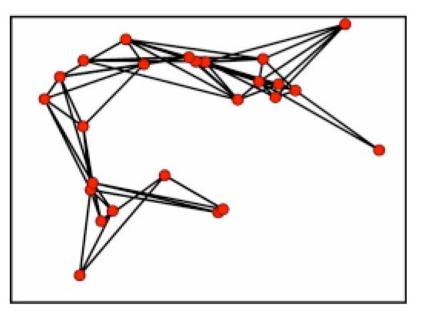
They rule: http://www.theyrule.net/2004/tr2.php





### Social Network

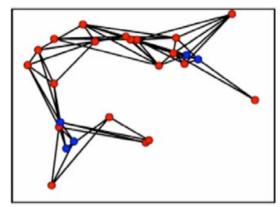
- $\equiv$  Freeman 2005 (Example taken from Spence 2007)
- $\equiv$  Employees of a department store spending leisure time together
- $\equiv$  Length of paths represents the shortest path between a pair of employees
- $\equiv$  What is the drive-force behind the pattern?



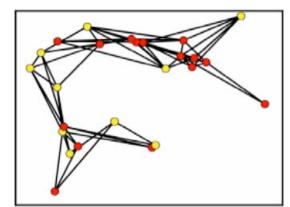


### **Social Network**

 $\equiv$  Color-code attributes to detect patterns



Middle-Eastern Ethnic Background



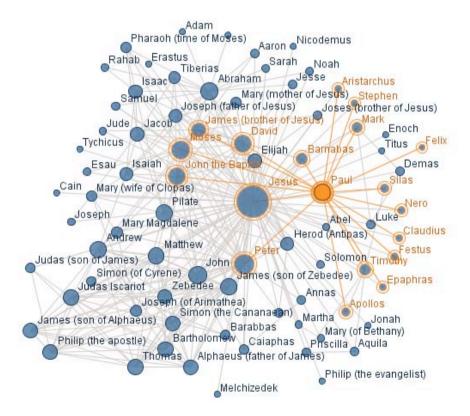
Married persons

Actor's Age Grades



### Social Network?

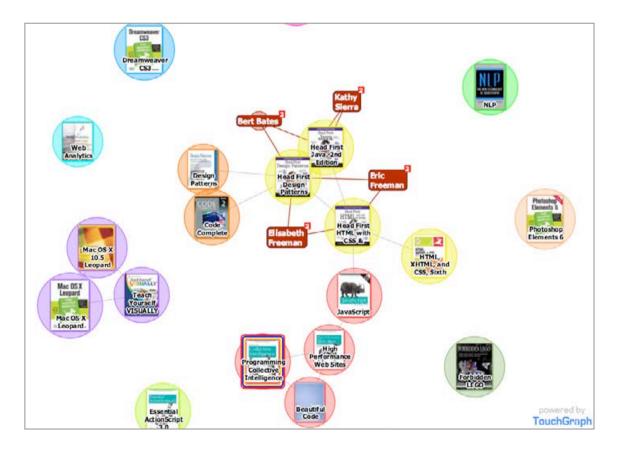
Co-occurrences of names in the new testament: http://services.alphaworks.ibm.com/manyeyes/view/SMGTJEsOtha6GEktsYeKE2-





## Copurchase Network

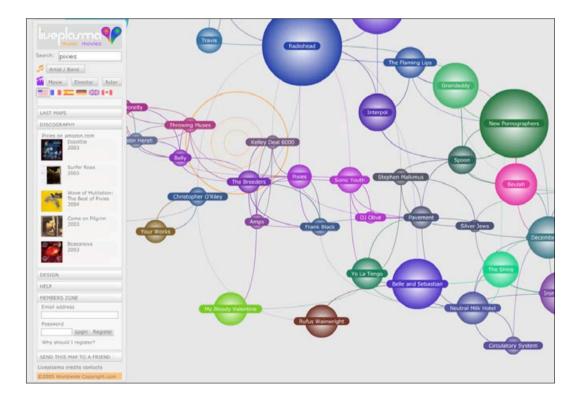
Touch graph: http://www.touchgraph.com/TGAmazonBrowser.html





## Music + Movie Network

- Liveplasma: http://www.liveplasma.com/
- $\equiv$  Mapping and data source unclear

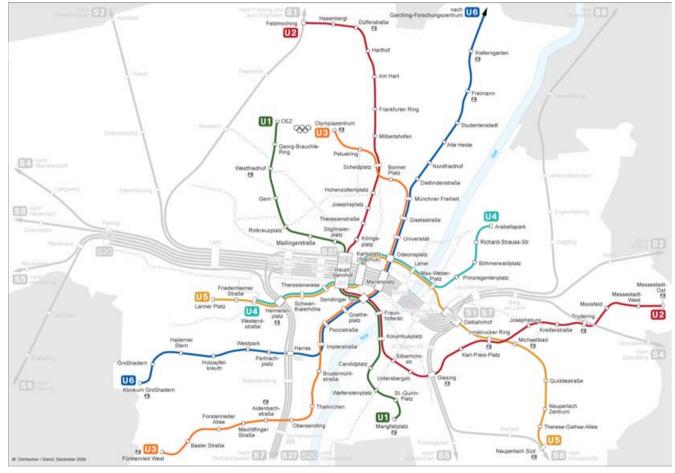


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#### **Transportation Network**



#### http://de.wikipedia.org/wiki/U-Bahn\_M%C3%BCnchen

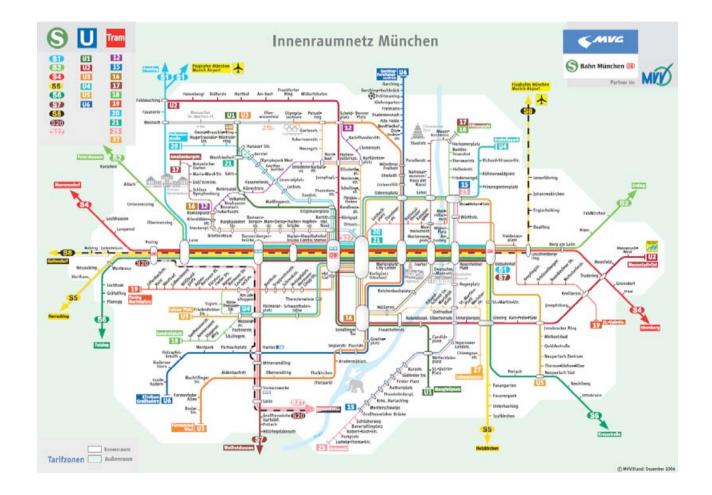
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#### **Transportation Network**





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## **Transportation Network**

#### Objectives

- $\equiv$  Facilitate understanding of network connections
- $\equiv$  Fit size and aspect ratio constraint (positioned above the doors in the underground )
- Heavily distorted geographic positions, but still good readability for identifying shortest paths between stations
- $\equiv$  Despite landmarks such as rivers, more graph than map





http://de.wikipedia.org/wiki/U-Bahn\_M%C3%BCnchen



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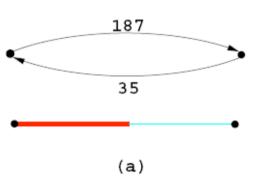
**Telephone Network** 

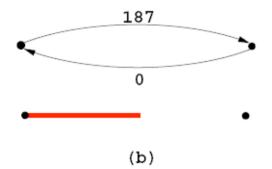
- 📃 Becker et al. 1995 AT&T data
- $\equiv$  110 switches (nearly) completely connected
  - $\equiv$  Each vertex has a geographic location
  - $\equiv$  Statistics for each vertex, new data every five minutes
- $\equiv$  12,000 links between switches
- October 17, 1989 earthquake in San Francisco Bay area
- $\equiv$  Questions related to network capacity and traffic flows
  - $\equiv$  Where are the overloads?
  - $\equiv$  Which links are carrying the most traffic?
  - $\equiv$  Was there network damage?
  - $\equiv$  Are there any pockets for underutilized network capacity?
  - $\equiv$  Is the overload increasing or decreasing?
  - $\equiv$  Are calls into the affected area completing or are they being blocked elsewhere in the network?
- $\equiv$  Different representations: linkmap, nodemap, matrix display



# Linkmap Encoding

- Switches (vertices) are arranged according to their geographical position
- Two-tiled edges represent overload of in- and outgoing calls between switches
- Redundant coding to make the important edges more apparent: color and line-thickness both indicate amount of overload
- Reduce clutter by omitting edge segments where the overload value is zero

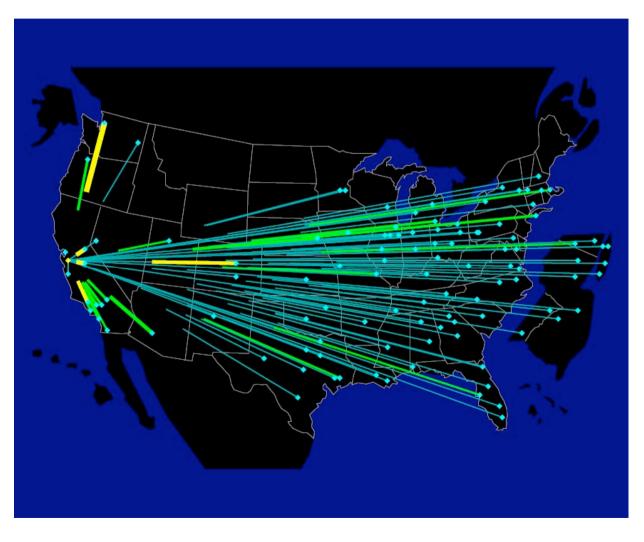






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### Linkmap - Oakland Switch



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# Linkmap - Oakland Switch

- $\equiv$  Overload into one switch
  - Into Oakland switch from every other node (most heavily from Seattle and Denver)
  - $\equiv$  Out of Oakland switch to many switches particularly on the east coast
- ➡ Island in the Atlantic Ocean is a blow-up of NY / New Jersey area (to reduce density of switches)
- $\equiv$  Does work well because the edges hardly overlap
- $\equiv$  What about showing total overload?

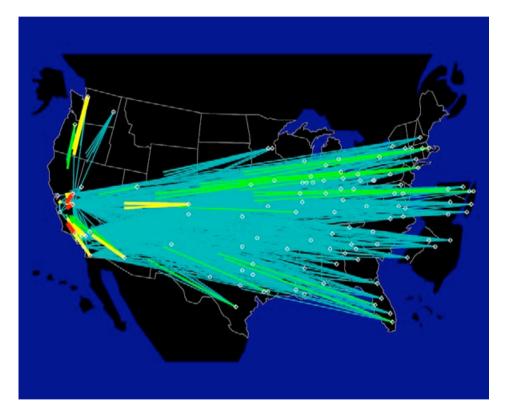




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# Linkmap - Total Overload

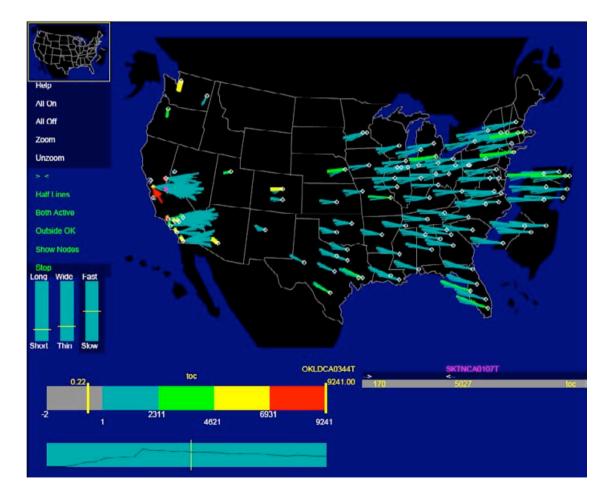
- $\equiv$  Most important links are drawn last
- Still: display is ineffective because long edges from one coast to another obscure much of the country
- To reduce clutter: edge may be drawn only part way between the vertices they connect





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## Linkmap - Total Overload





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### Nodemap

#### ■ Glyph encoding

- $\equiv$  Aggregate overload into and out of each switch
- Rectangle width: proportional to the square root of the number of incoming calls
- Rectangle height: proportional to the square root of the number of outgoing calls
- Area of rectangle proportional to total overload
- Interpretation: overload of outgoing calls from nodes to northern and southern California
- $\equiv$  Problem with this kind of representation?
- No clutter, but detailed information about particular links between switches is lost

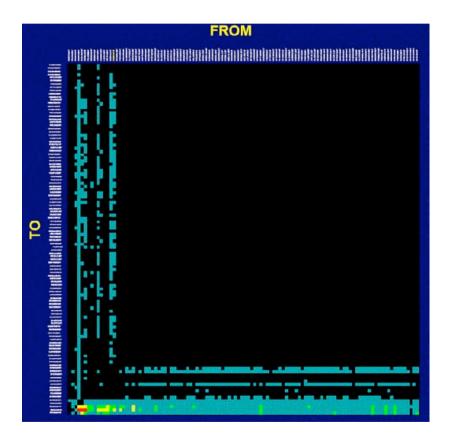




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# Matrix Display

- $\equiv$  Omits information about geography
- Each matrix element is allocated to a directed link (half-line)
- Each switch is assigned to one row (incoming calls) and one column (outgoing calls)
- $\equiv$  Switches are arranged west-to-east
- Interpretation
  - $\equiv$  Five switches with major incoming overload (rows)
  - One switch with outgoing overload to almost every other node (column)
- $\equiv$  Very compact visualization without clutter
- $\equiv$  Problems with this kind of representation?
- Inference of the visualization is influenced by the ordering of the rows and columns
- Intuitiveness and readability when compared to a node-link diagram?





Outline

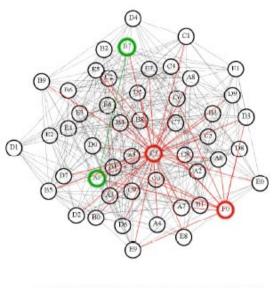
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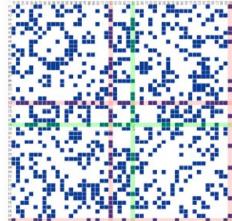


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## Node-link versus Matrix

- Ghoniem et al. 2004
- $\equiv$  On-demand highlighting of selected nodes and links
- 36 participants
- $\equiv$  Tasks to test readability
  - $\equiv$  Estimation of number of vertices in the graph
  - $\equiv$  Estimation of number of edges
  - $\equiv$  Locating most connected node
  - $\equiv$  Locate node by label
  - $\equiv$  Find link between two specified nodes
  - $\equiv$  Finding a common neighbor between two specified nodes
  - $\equiv$  Finding a path between two nodes
- Random undirected graphs of three different sizes (number of vertices) and density (relative number of edges)



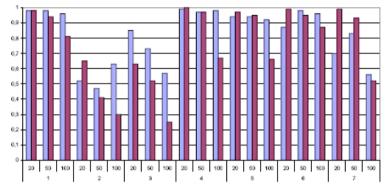




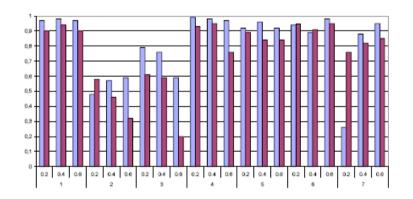
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## Node-link versus Matrix

- Independent variables
  - **∃** Graph representation
  - $\equiv$  Number of vertices
  - $\equiv$  Relative number of edges
- $\equiv$  Dependent variables
  - $\equiv$  Answer time (results not shown here)
  - $\equiv$  Number of correct answers
- All users were familiar with node-link diagrams, but not with matrices
- Node-link diagrams seem to be well suited for small graphs but their readability quickly deteriorates with a growing size of the graph and link density
- Matrix provides a superior readability for large or dense graphs
- $\equiv$  Node-link diagram only clearly superior for find-path task



*Figure 2* Percentage of correct answers split by task and by size. The matrix representation appears in blue and the node-link in purple.



*Figure 3* Percentage of correct answers split by task and by density. The matrix representation appears in blue and the node-link in purple.



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## **Graph Interaction**

- Dynamic visualization & interaction is essential for exploring / navigating graphs
  - $\equiv$  Dragging and highlighting of vertices and edges
  - Filtering
  - ∃ Zooming & panning
  - Focus+context distortion
- $\equiv$  Animation can support exploration



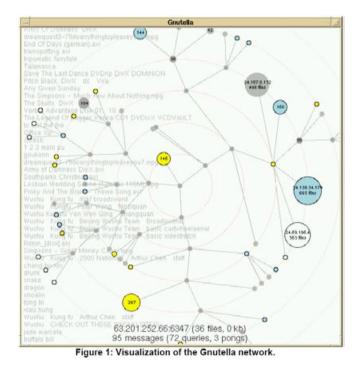
Focus+Context graph - Jankun-Kelly et al. 2003



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#### **Transitions in Radial Tree Layout**

- Radial tree layout: common technique in which the graph is arranged around a focus node
- Users can change the layout by selecting a different focus node
- $\equiv$  Animated transitions of node translation
- $\equiv$  Objective: keep the transitions easy to follow
- $\equiv$  Animation mechanism
  - $\equiv$  Linear interpolation of polar coordinates of the nodes
  - $\equiv$  Follows ordering and orientation constraints
- Movie 📃







# **Additional Sources and Literature**

#### $\equiv$ Obligatory reading

- Nathalie Henry, Jean-Daniel Fekete, and Michael J. McGuffin: "NodeTrix: A Hybrid Visualization of Social Networks", InfoVis, 2007.
- http://insitu.lri.fr/~nhenry/docs/Henry-InfoVis2007.pdf
- $\equiv$  Tutorials for graph theory and graph drawing
  - http://www.cs.usask.ca/resources/tutorials/csconcepts/1999\_8/
  - ∃ http://davis.wpi.edu/~matt/courses/graphs/