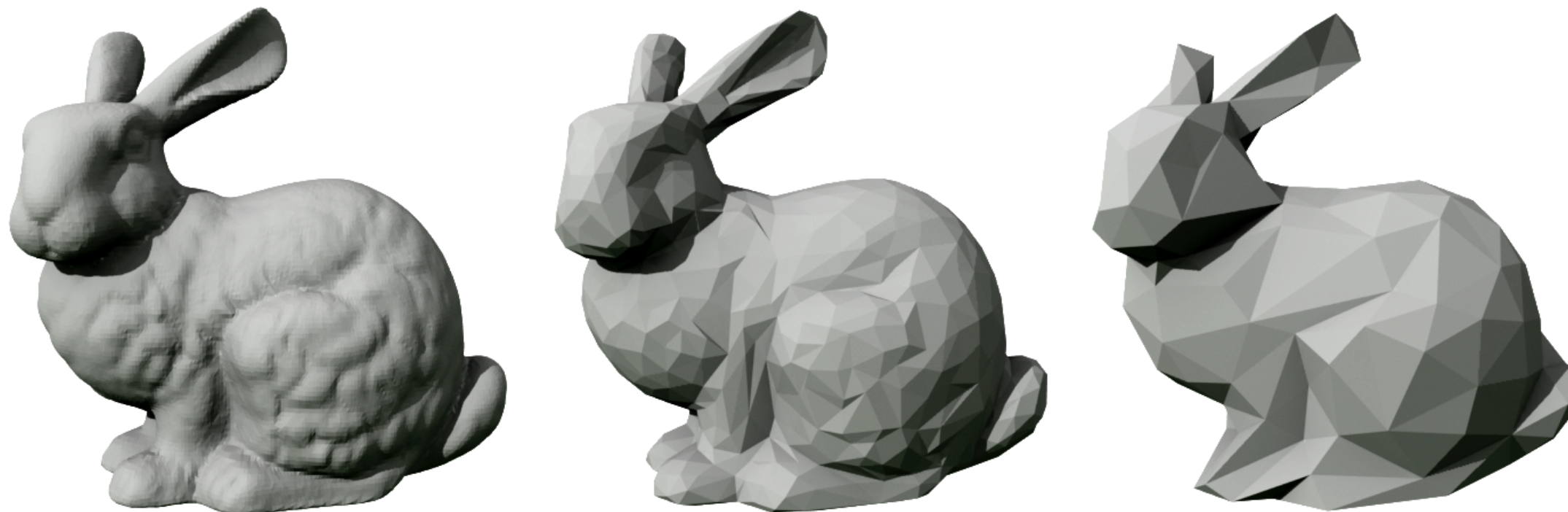


Computer Graphics 1

Ludwig-Maximilians-Universität München
Summer semester 2020

Prof. Dr.-Ing. Andreas Butz

lecture additions by Dr. Michael Krone, Univ. Stuttgart



https://commons.wikimedia.org/wiki/File:Stanford_bunny_qem.png

Sources

- This lecture was introduced by Michael Krone and is based on the slides of Filip Sadlo for the lecture „*Visualization in Science and Engineering*“
- Course slides make use of selective contributions from
 - Thomas Ertl
 - Daniel Weiskopf
 - Carsten Dachsbacher
 - Oliver Deussen
 - Rüdiger Westermann
 - Stefan Gumbold
 - Dirk Bartz
 - Torsten Möller
 - Ronald Peikert

Chapter 10 – Volume Rendering & Scalar Field Visualization

- Basic strategies

- Function plots and height fields
- Isolines
- Color coding

- Volume data

- Overview of volume visualization approaches
- Slicing
- Indirect volume visualization
- Direct volume rendering
- Classification and segmentation

Basic Strategies

- Visualization of 1D, 2D, or 3D scalar fields

- 1D scalar field: $\Omega \subset \mathbb{R} \rightarrow \mathbb{R}$

- 2D scalar field: $\Omega \subset \mathbb{R}^2 \rightarrow \mathbb{R}$

- 3D scalar field: $\Omega \subset \mathbb{R}^3 \rightarrow \mathbb{R}$ → Volume visualization

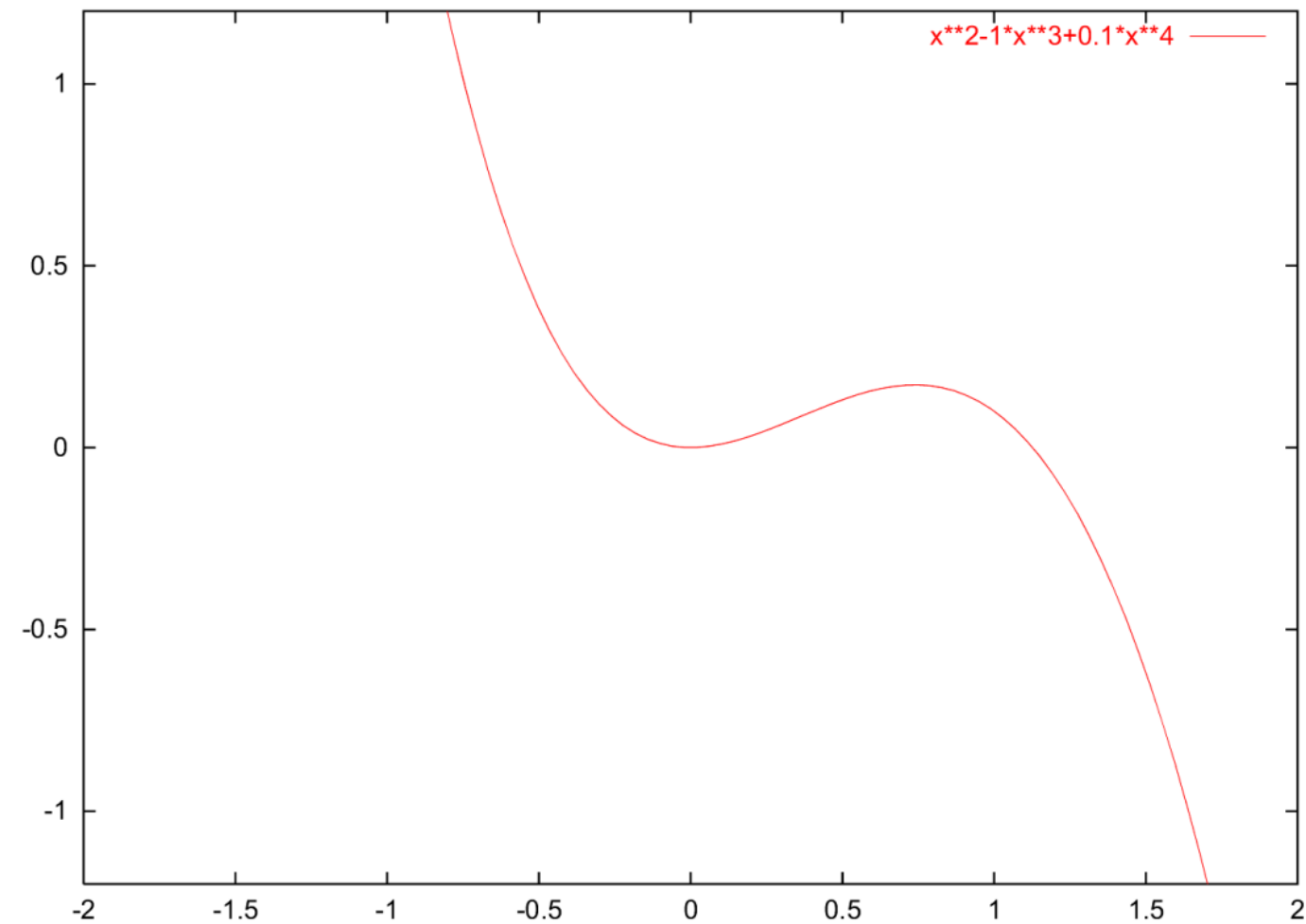
Basic Strategies

- Mapping to geometry
 - Function plots
 - Height fields
 - Isolines and isosurfaces
- Color coding
- Specific techniques for 3D data
 - Indirect volume visualization
 - Direct volume visualization

→ Visualization method depends heavily on dimensionality of domain

Function Plots and Height Fields

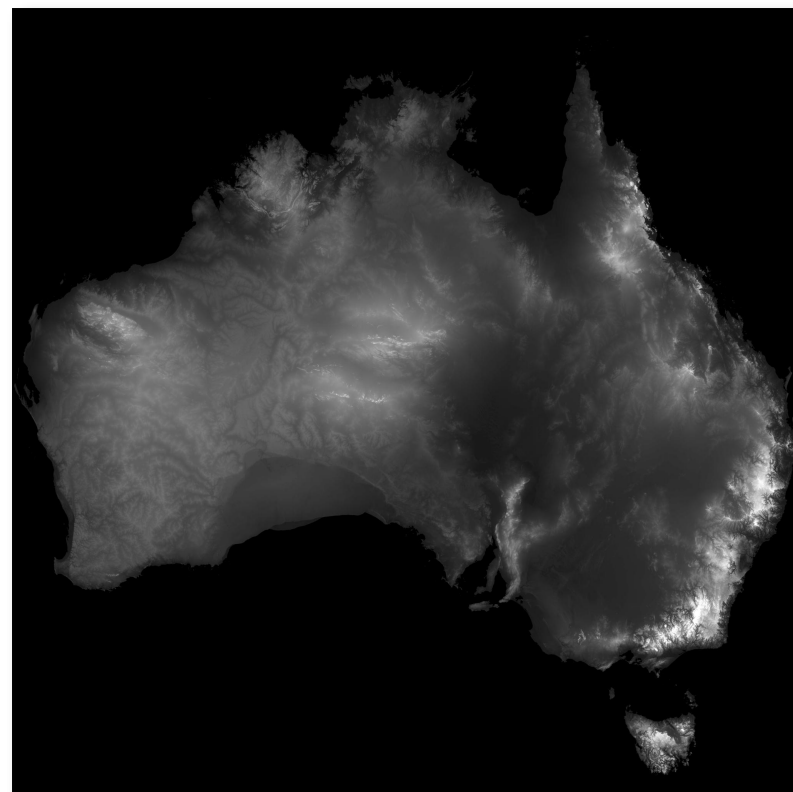
- Function plot for a 1D scalar field
 - Points $\{(s, f(s)) \mid s \in \mathbb{R}\}$
 - 1D manifold: line
 - Error bars possible



Gnuplot example

Function Plots and Height Fields

- Function plot for a 2D scalar field
 - Points $\{(s,t,f(s,t)) \mid (s,t) \in \mathbb{R}^2\}$
 - 2D manifold: surface
- Surface representations
 - Wireframe
 - Hidden lines
 - Shaded surface

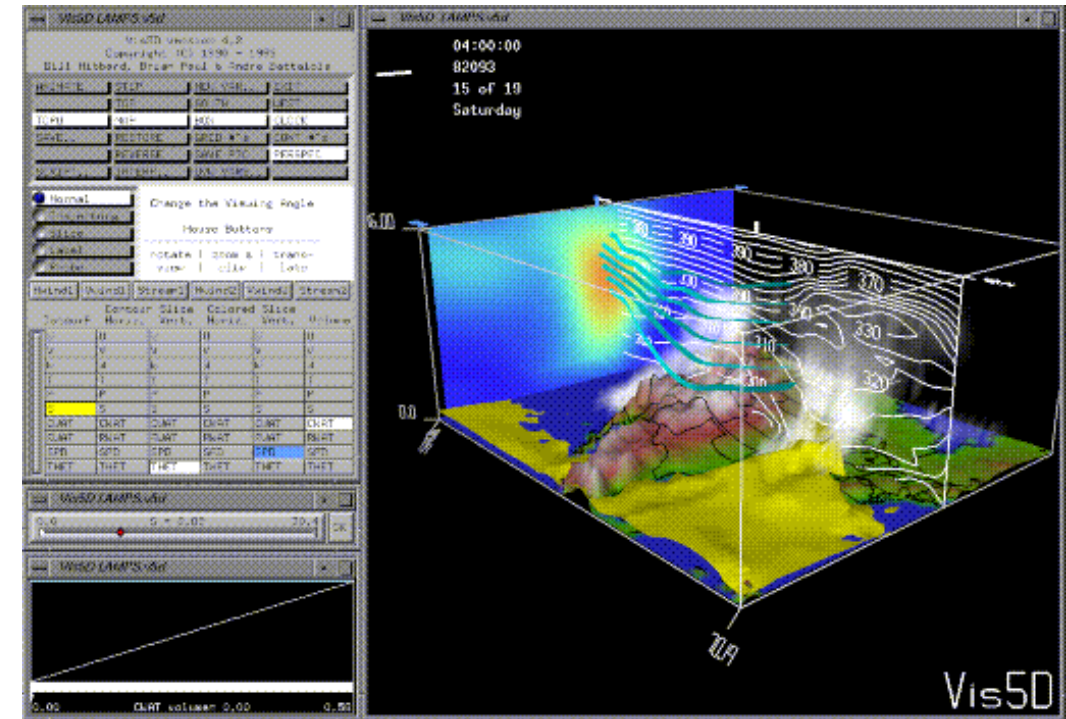


Isolines

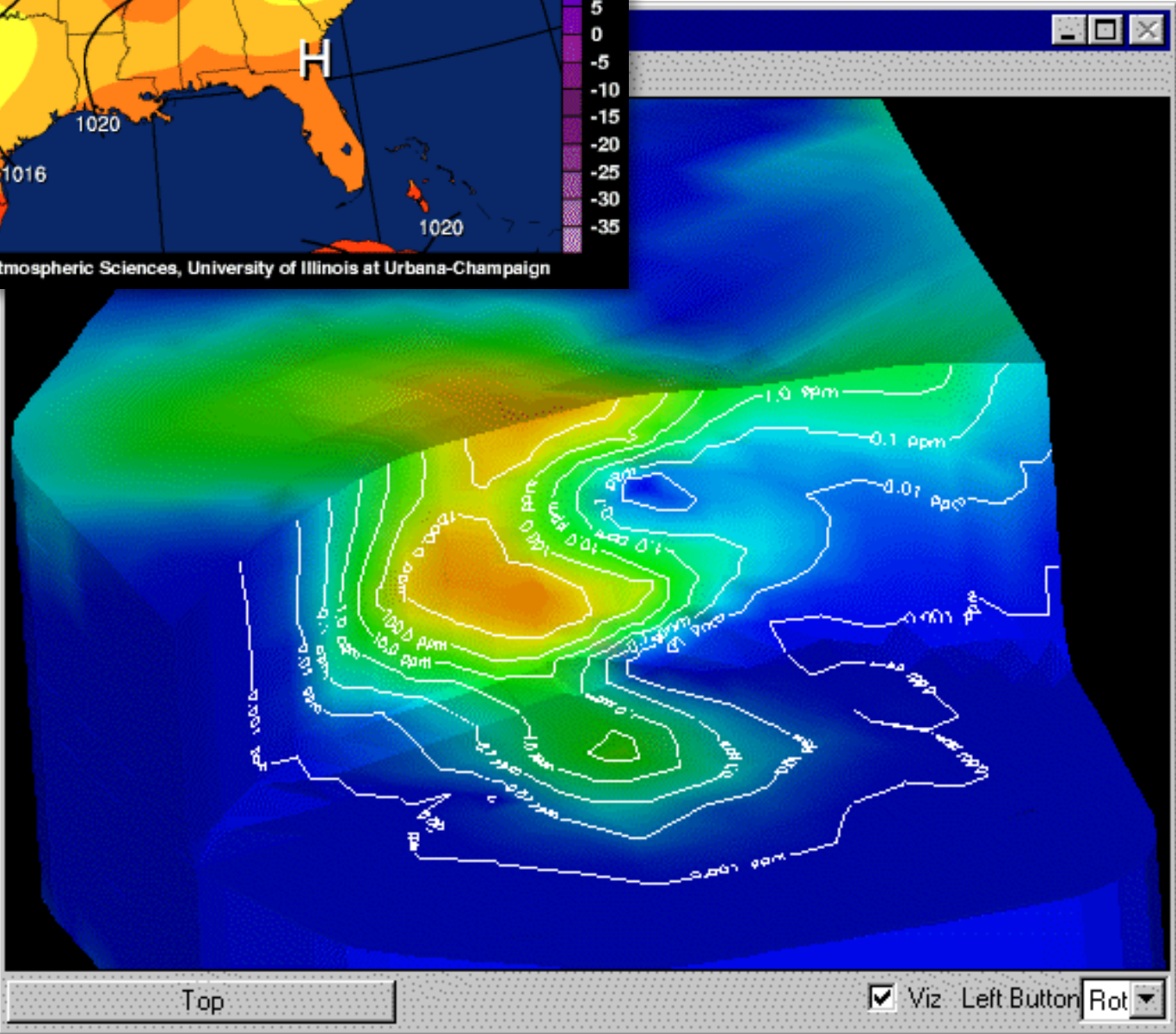
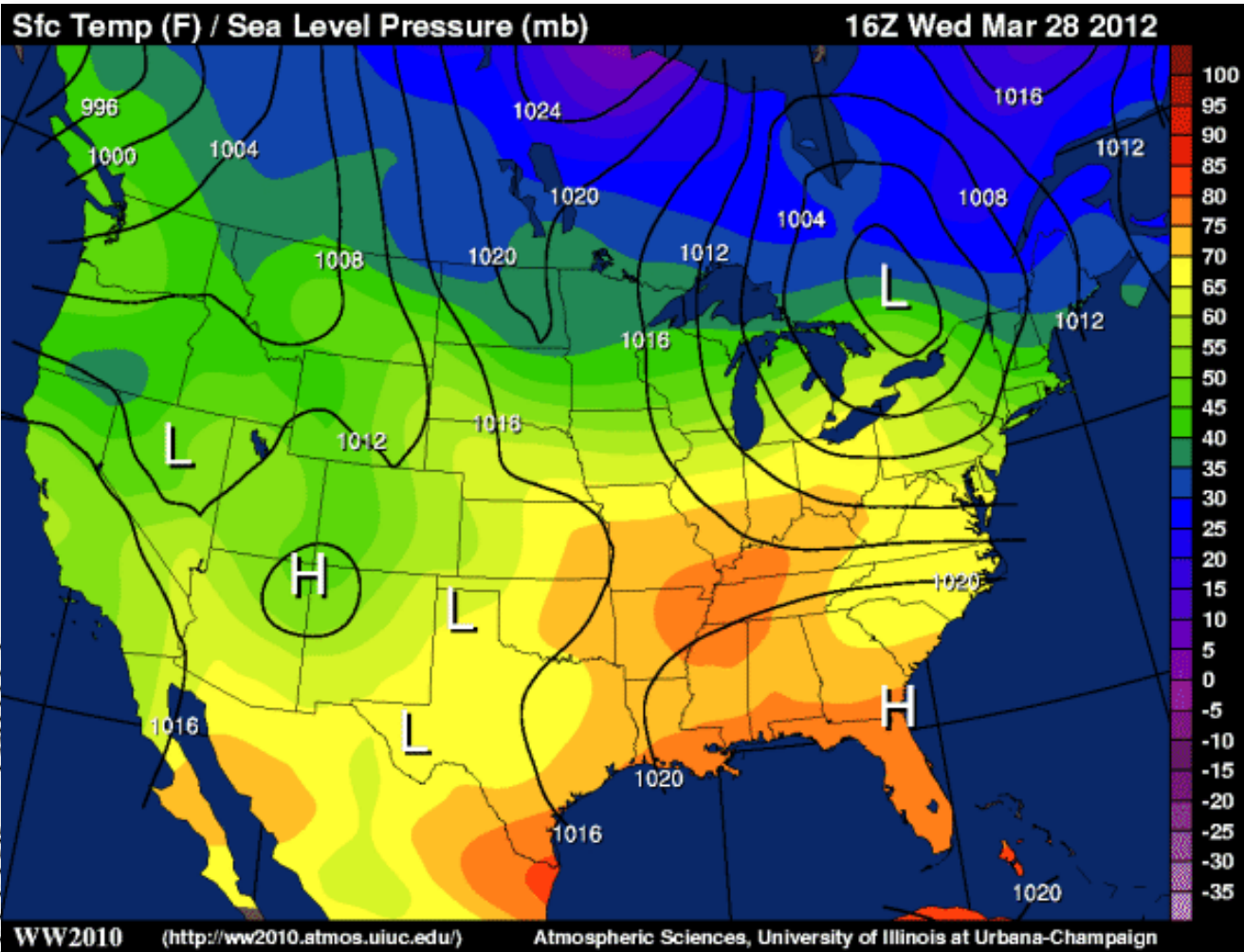
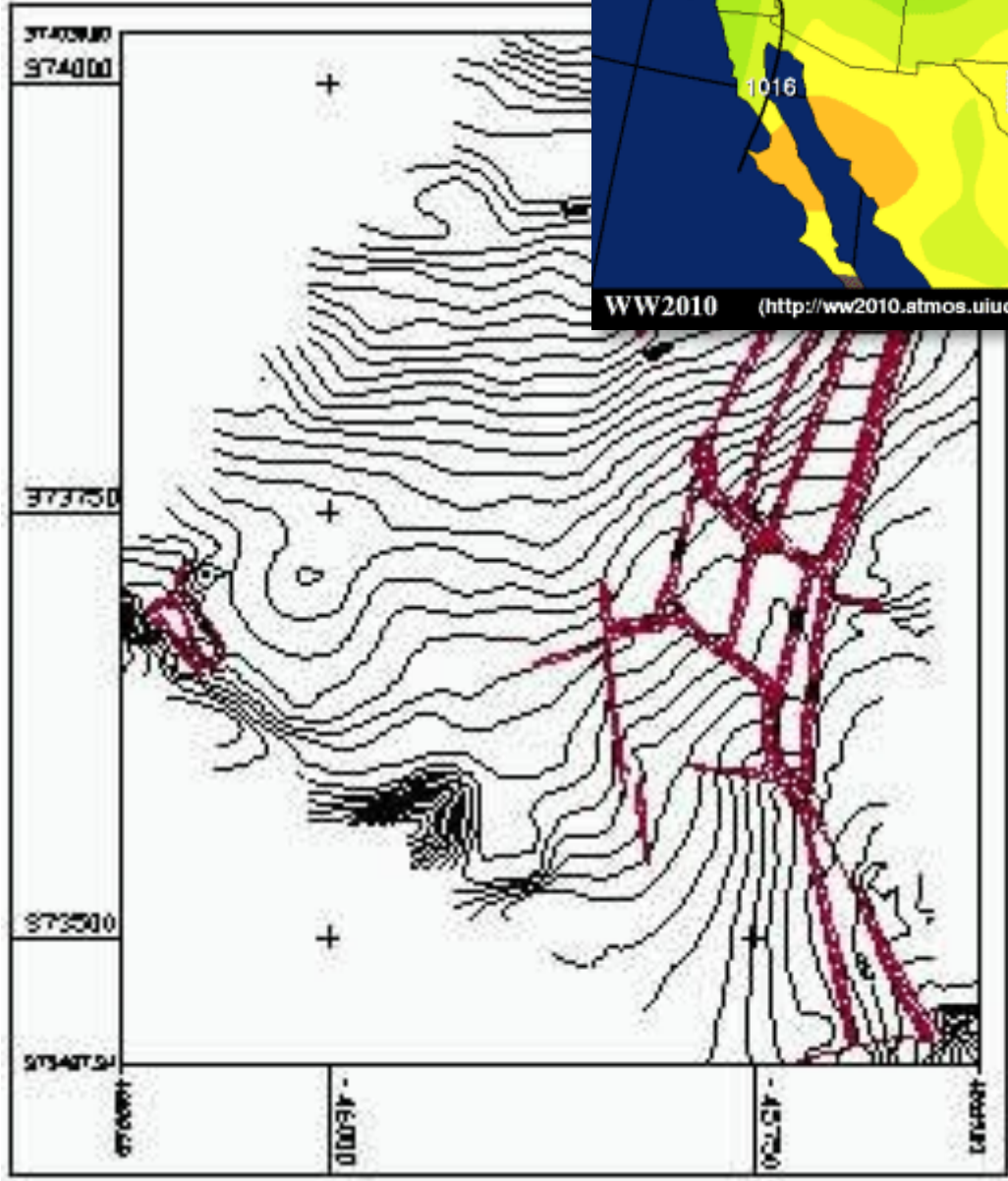
- Visualization of 2D scalar fields
- Given a scalar function $f : \Omega \rightarrow \mathbb{R}$ and a scalar value (isovalue) $c \in \mathbb{R}$
- Isoline consists of points

$$\{(x, y) \mid f(x, y) = c\}$$

- If $f()$ is differentiable and $\text{grad}(f) \neq 0$, then isolines are curves
- Contour lines



Isolines



Isolines: Pixel-by-Pixel Contouring

- Straightforward approach: scanning all pixels for equivalence with isovalue

- Input

- $f : (1, \dots, x_{max}) \times (1, \dots, y_{max}) \rightarrow R$
- Isovalues I_1, \dots, I_n and isocolors c_1, \dots, c_n

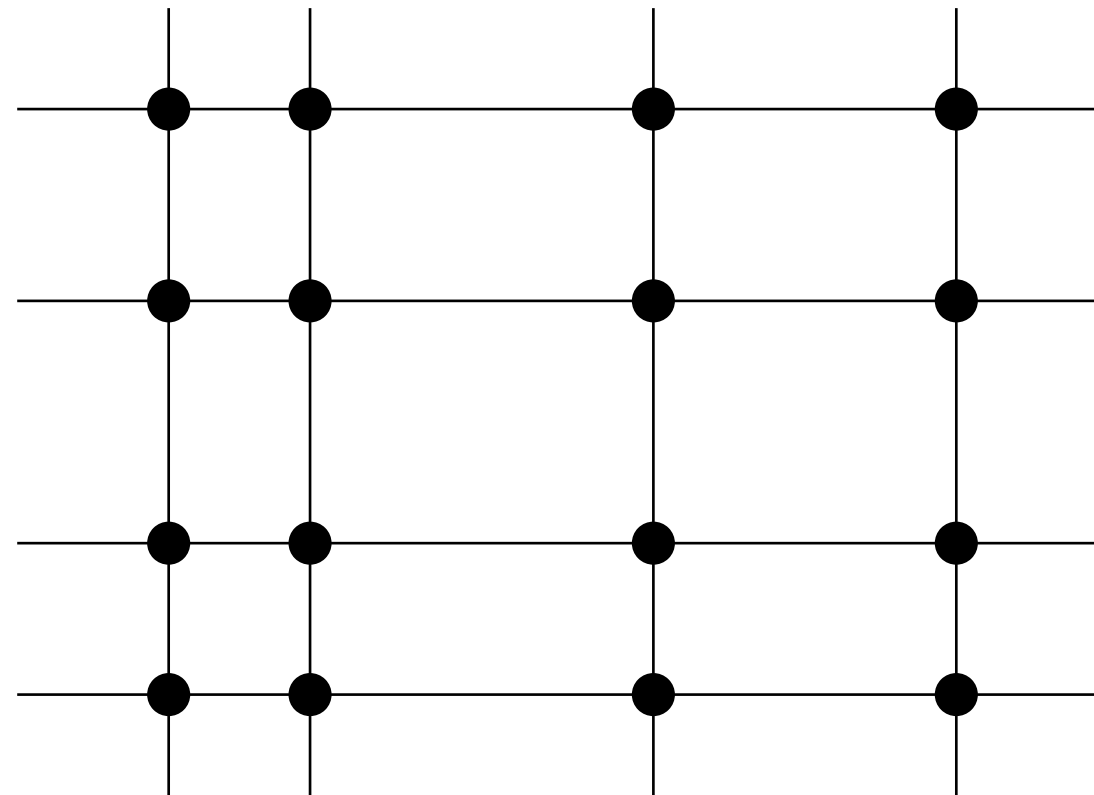
- Algorithm

```
for all  $(x, y) \in (1, \dots, x_{max}) \times (1, \dots, y_{max})$  do
  for all  $k \in \{ 1, \dots, n \}$  do
    if  $|f(x, y) - I_k| < \epsilon$  then
      draw( $x, y, c_k$ )
```

- Problem: Isoline can be missed if the gradient of $f()$ is too large (despite range ϵ)

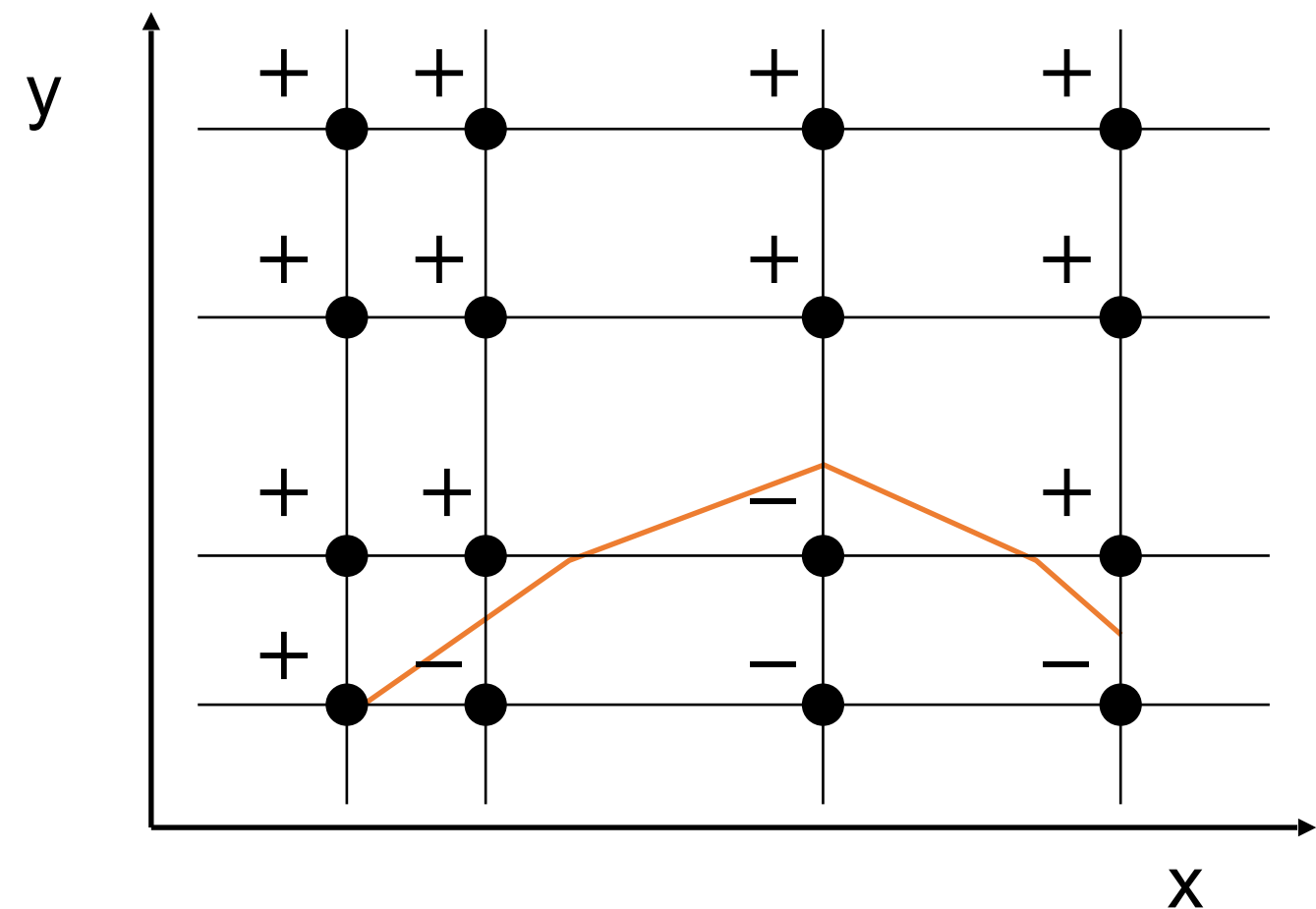
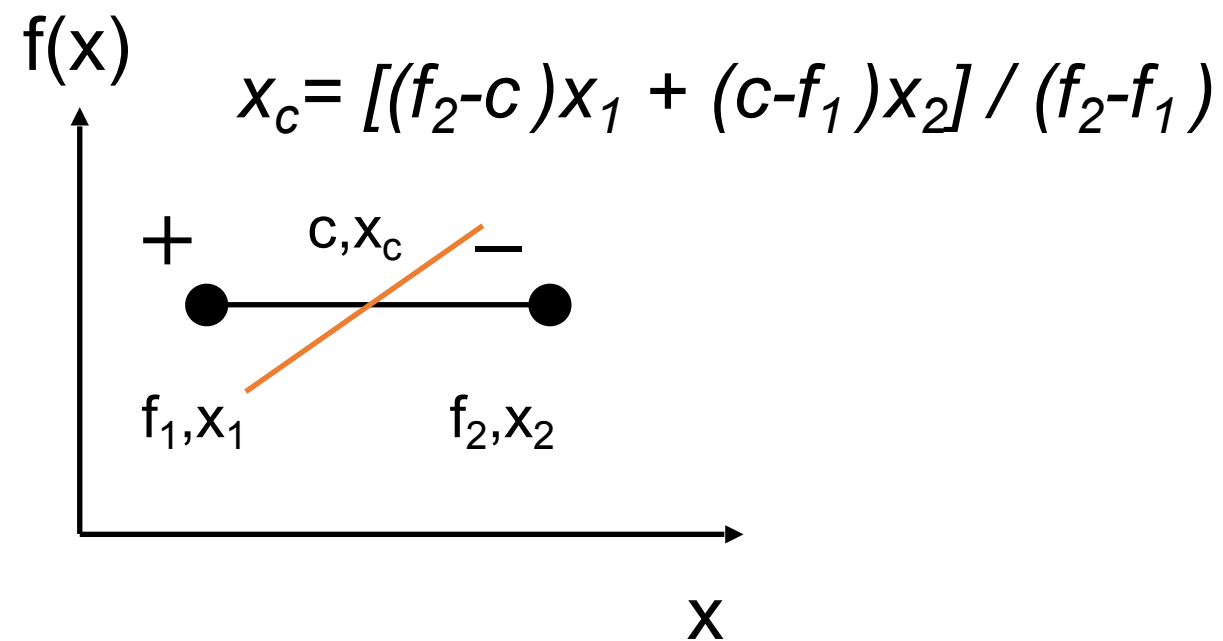
Isolines: Marching Squares

- Representation of the scalar function on a uniform or rectilinear grid
- Scalar values are given at each vertex $f \leftrightarrow f_{ij}$
- Take into account the interpolation within cells
- Consider cells independently of each other



Isolines: Marching Squares

- Which cells will be intersected ?
 - Initially mark all vertices by + or - , depending on the conditions $f_{ij} \geq c$, $f_{ij} < c$
- No isoline passes through cells (=rectangles) which have the same sign at all four vertices
 - So we only have to determine the edges with different signs
 - And find the intersection point by linear interpolation



Color Coding

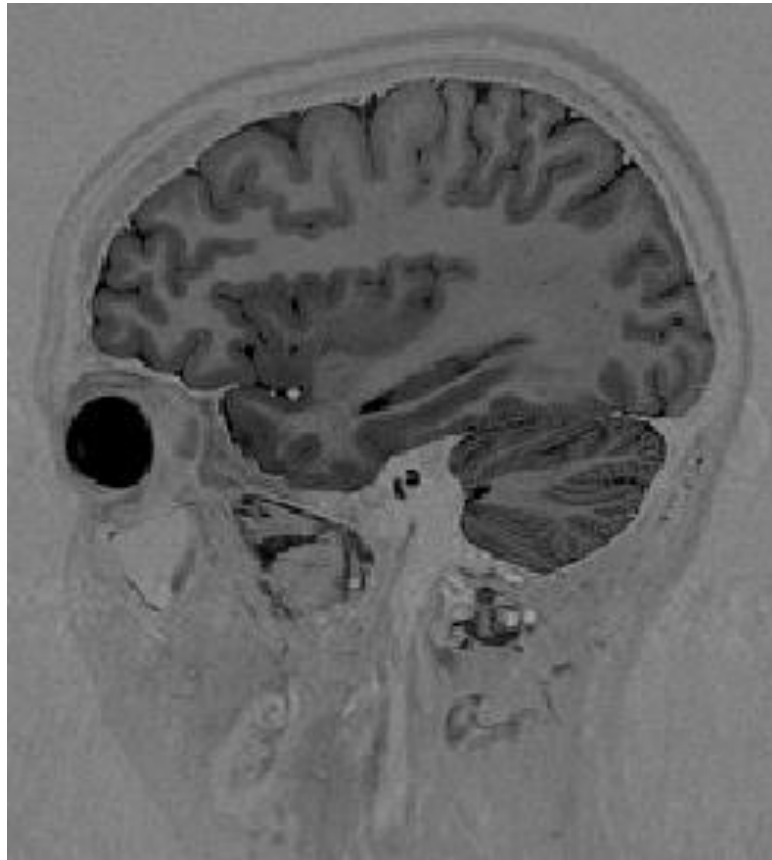
- Easy to apply to 1D and 2D scalar fields
 - Map color to each pixel on 1D or 2D image

$$\Omega \subset \mathbb{R}^2 \rightarrow \mathbb{R}^3$$

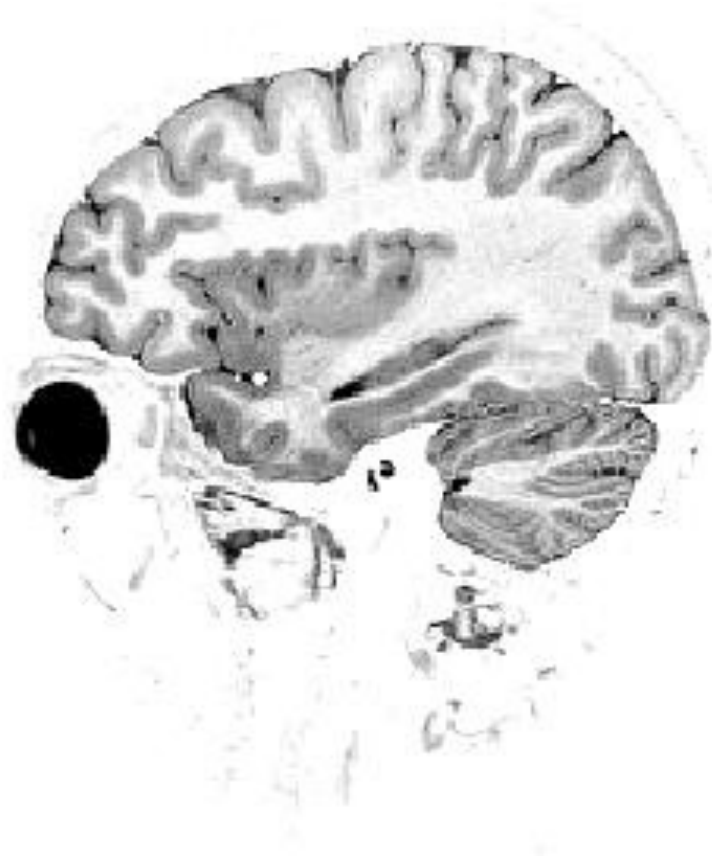


Color Coding

- Example: Medical images
 - Special color table to visualize the brain tissue
 - Special color table to visualize the bone structure



Original



Brain



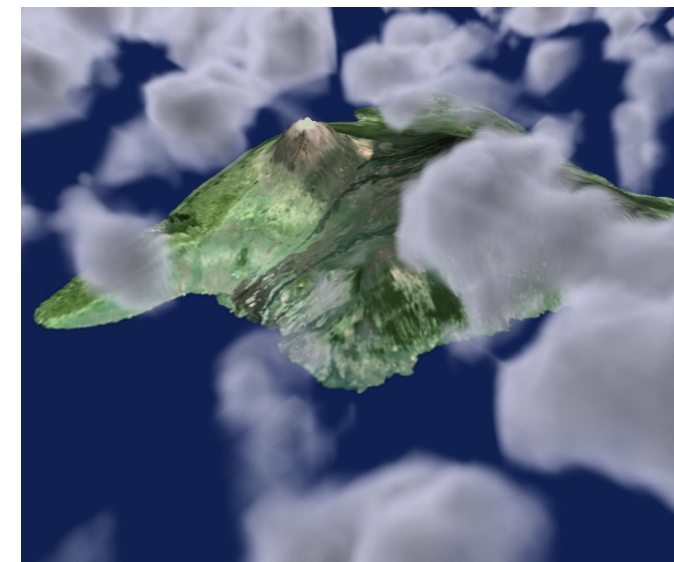
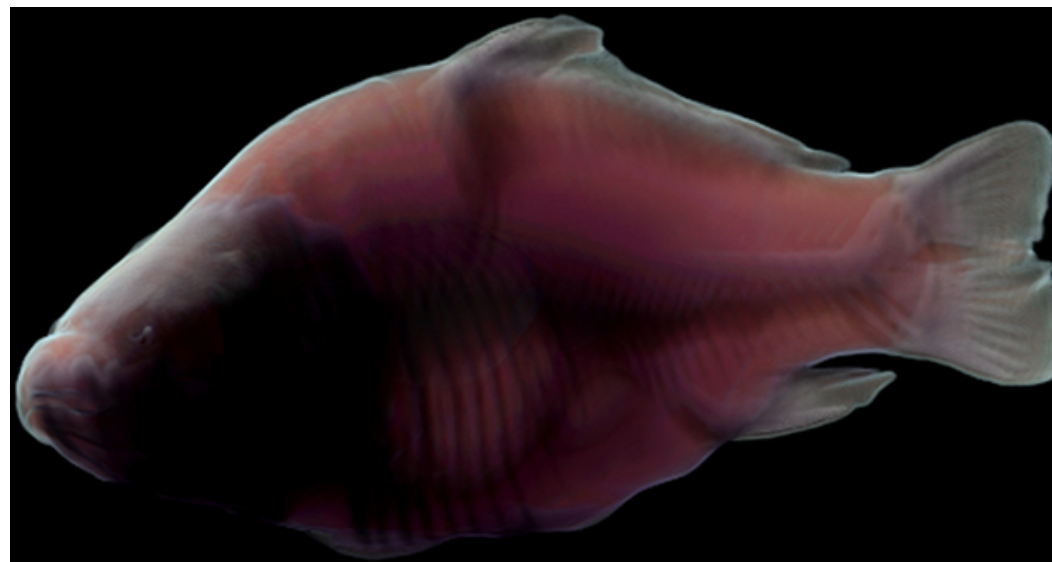
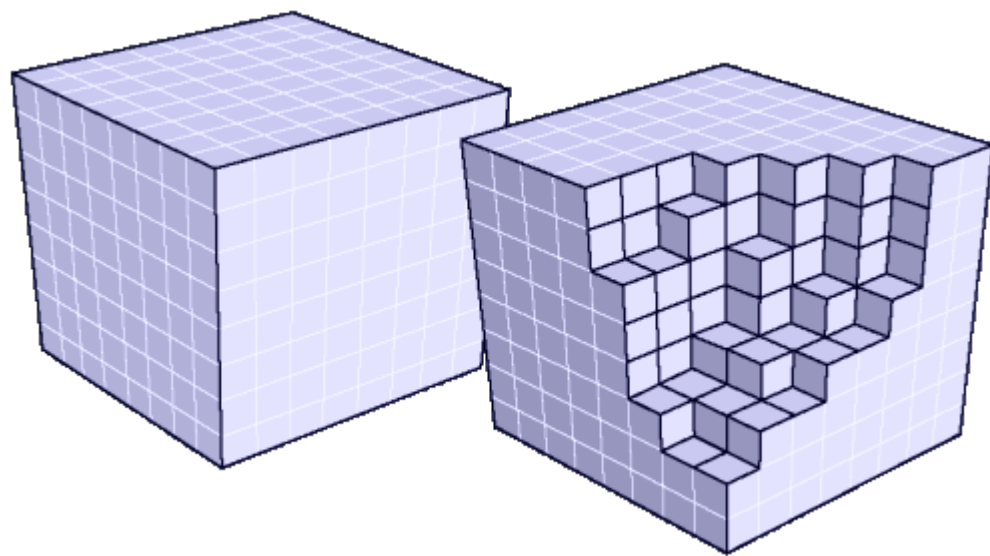
Tissue

Chapter 10 – Volume Rendering & Scalar Field Visualization

- **Basic strategies**
 - Function plots and height fields
 - Isolines
 - Color coding
- **Volume data**
 - Overview of volume visualization approaches
 - Slicing
 - Indirect volume visualization
 - Direct volume rendering
 - Classification and segmentation

Volume Data

- Simple case: regular, rectilinear 3D grid with cubic cells
 - Stores one or more values per grid cell
 - Grid cell = voxel (volume pixel)
- Data sources (examples)
 - Measurements, e.g., medical imaging (CT, MRT, 3D ultrasound...)
 - Simulation, e.g., fluid simulations (water, smoke, fog...)
 - Voxelization of 3D models, e.g., write closest distance to a surface to each voxel
 - Mathematical function



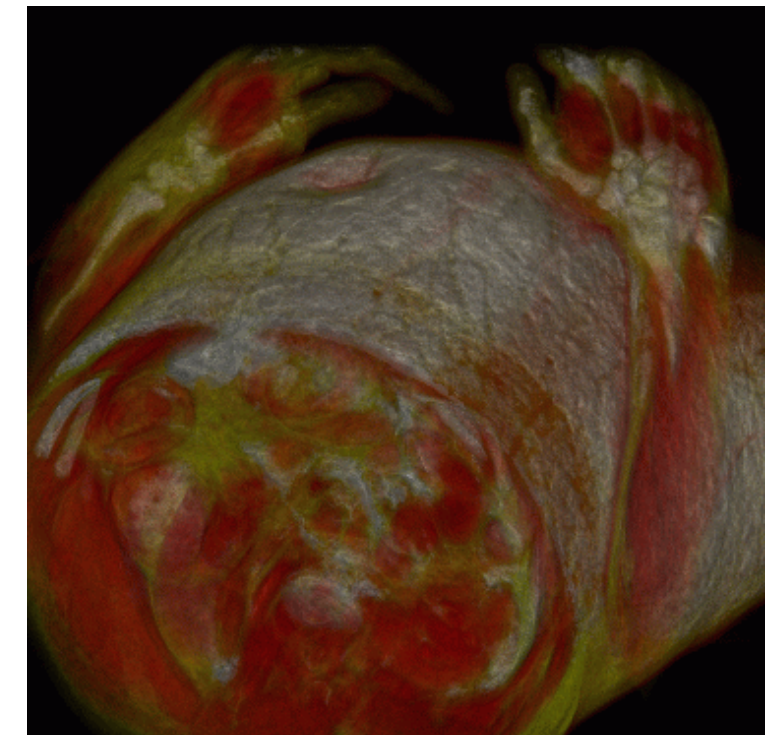
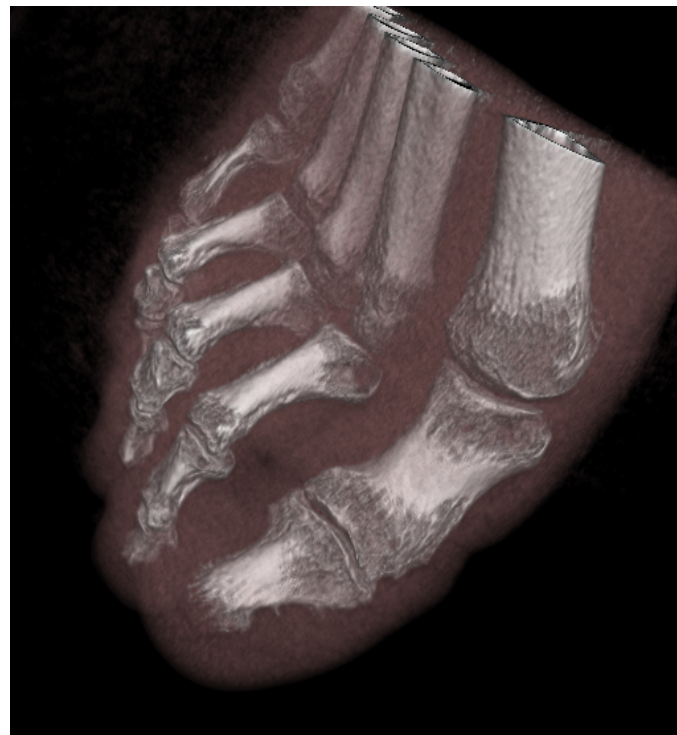
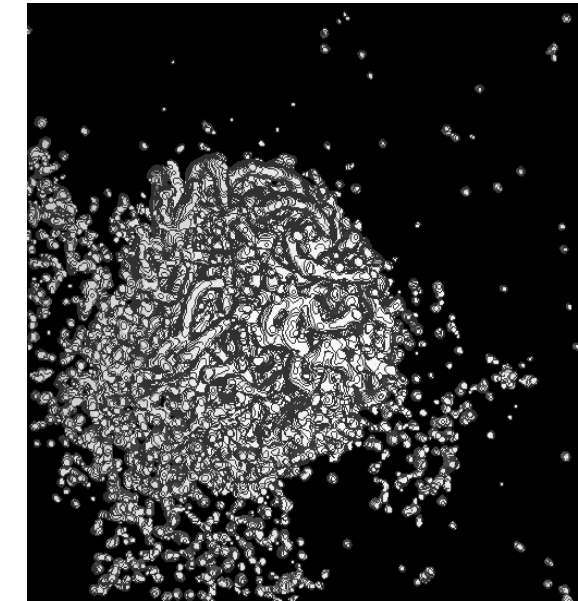
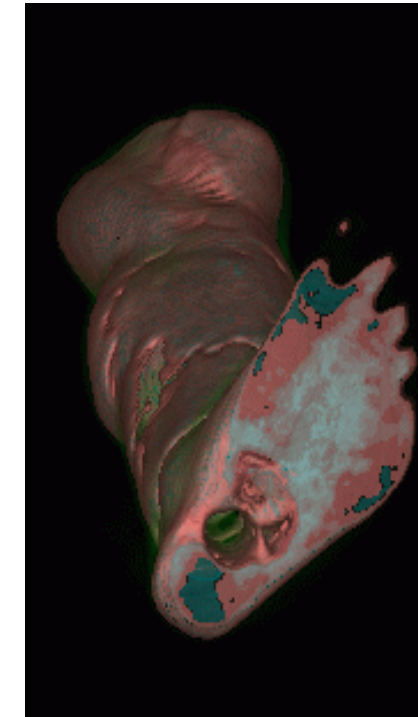
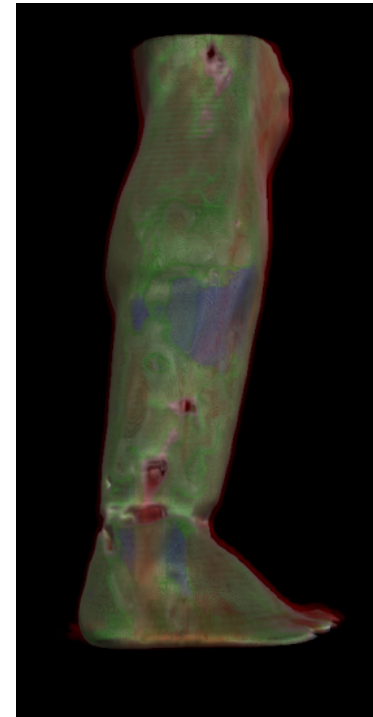
Volume Visualization

- Scalar volume data

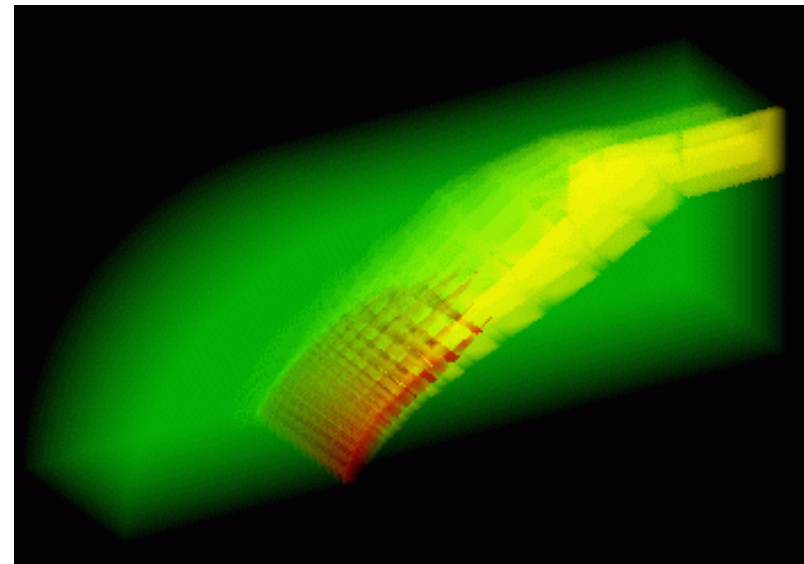
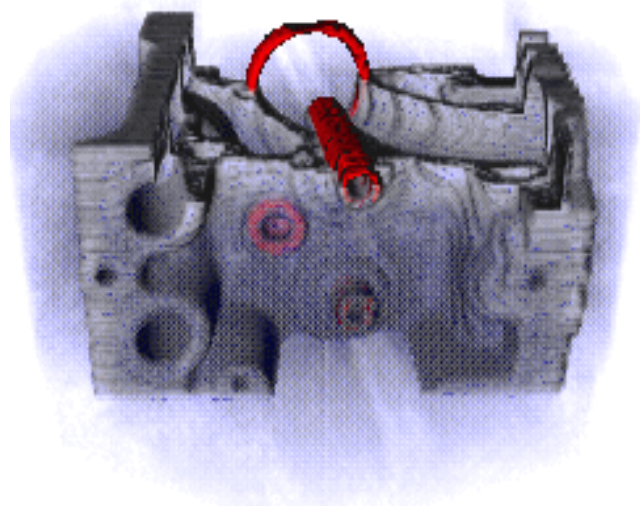
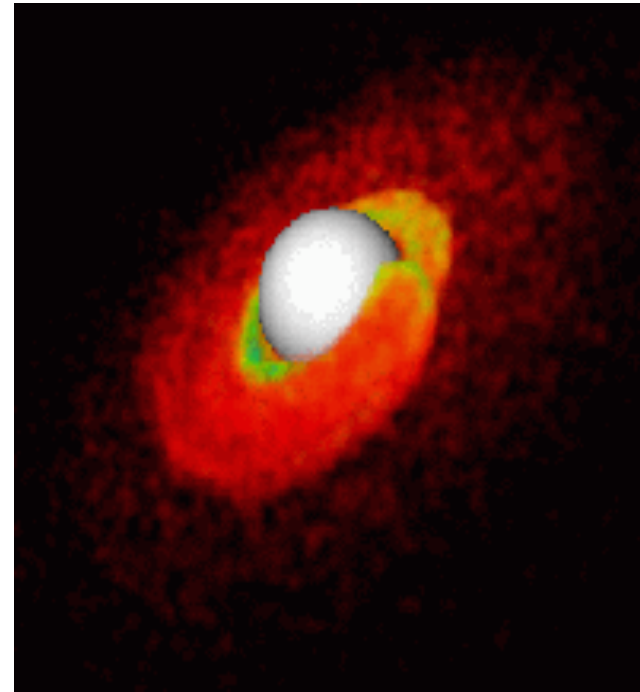
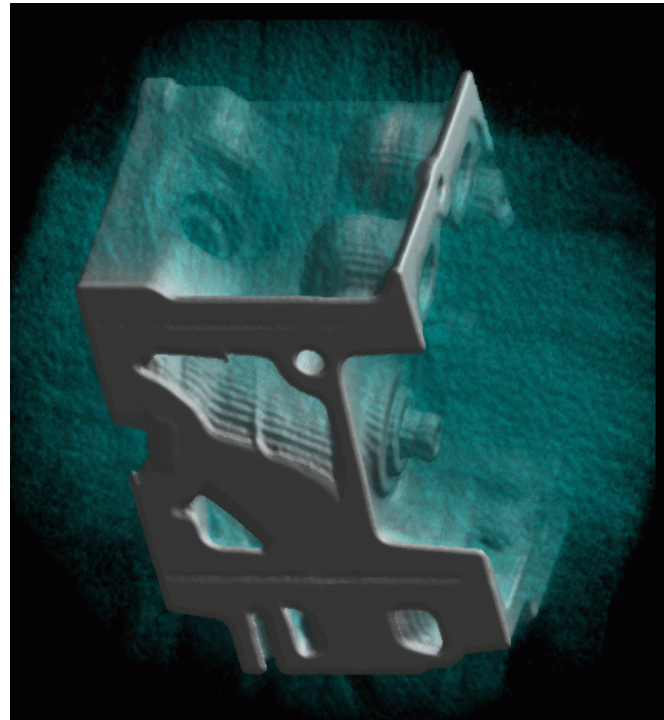
$$\Omega \subset \mathbb{R}^3 \rightarrow \mathbb{R}$$

- Medical Applications:

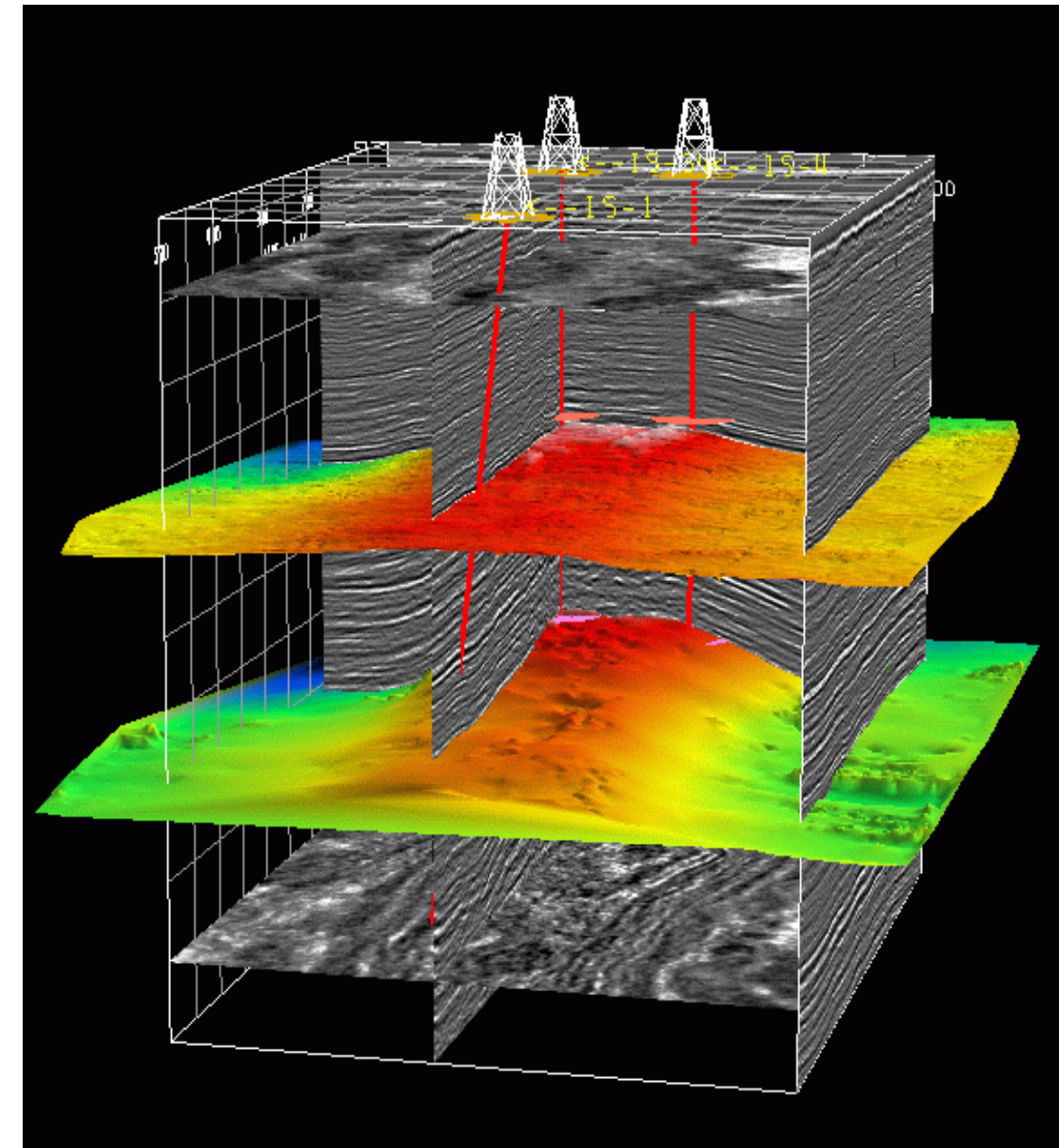
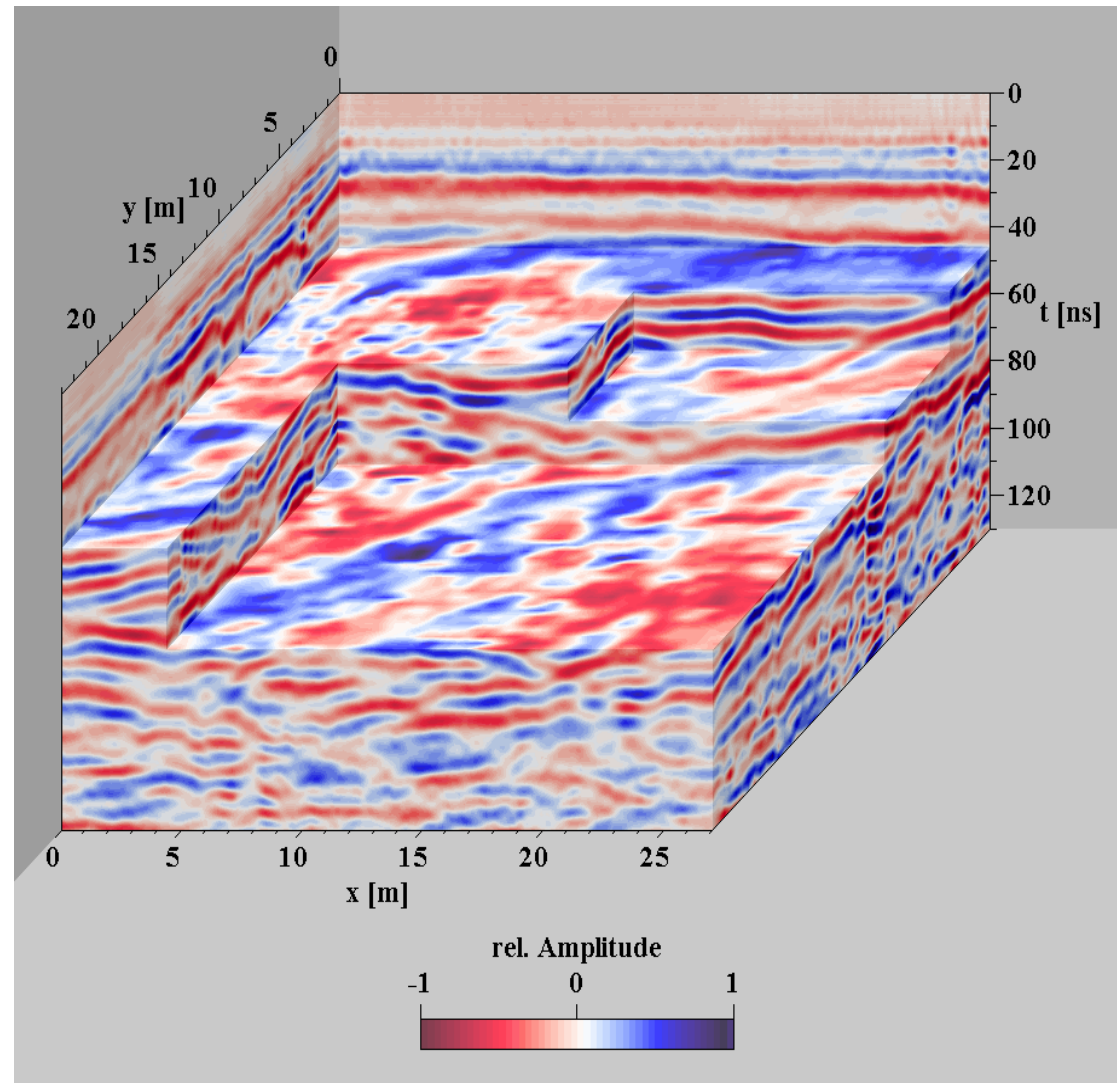
- CT, MRI, confocal microscopy, ultrasound, etc.



Volume Visualization



Volume Visualization

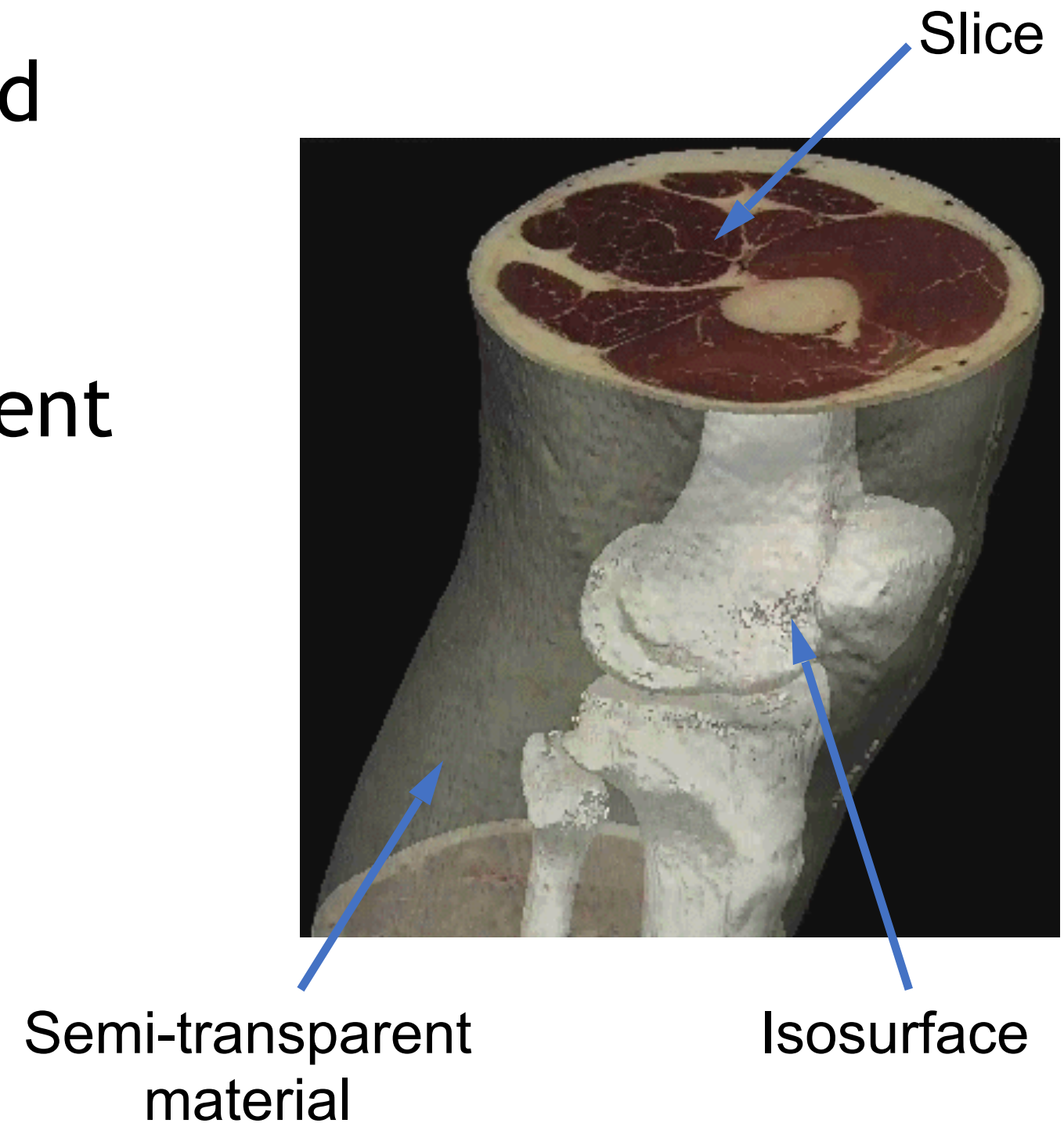


Volume Visualization Approaches

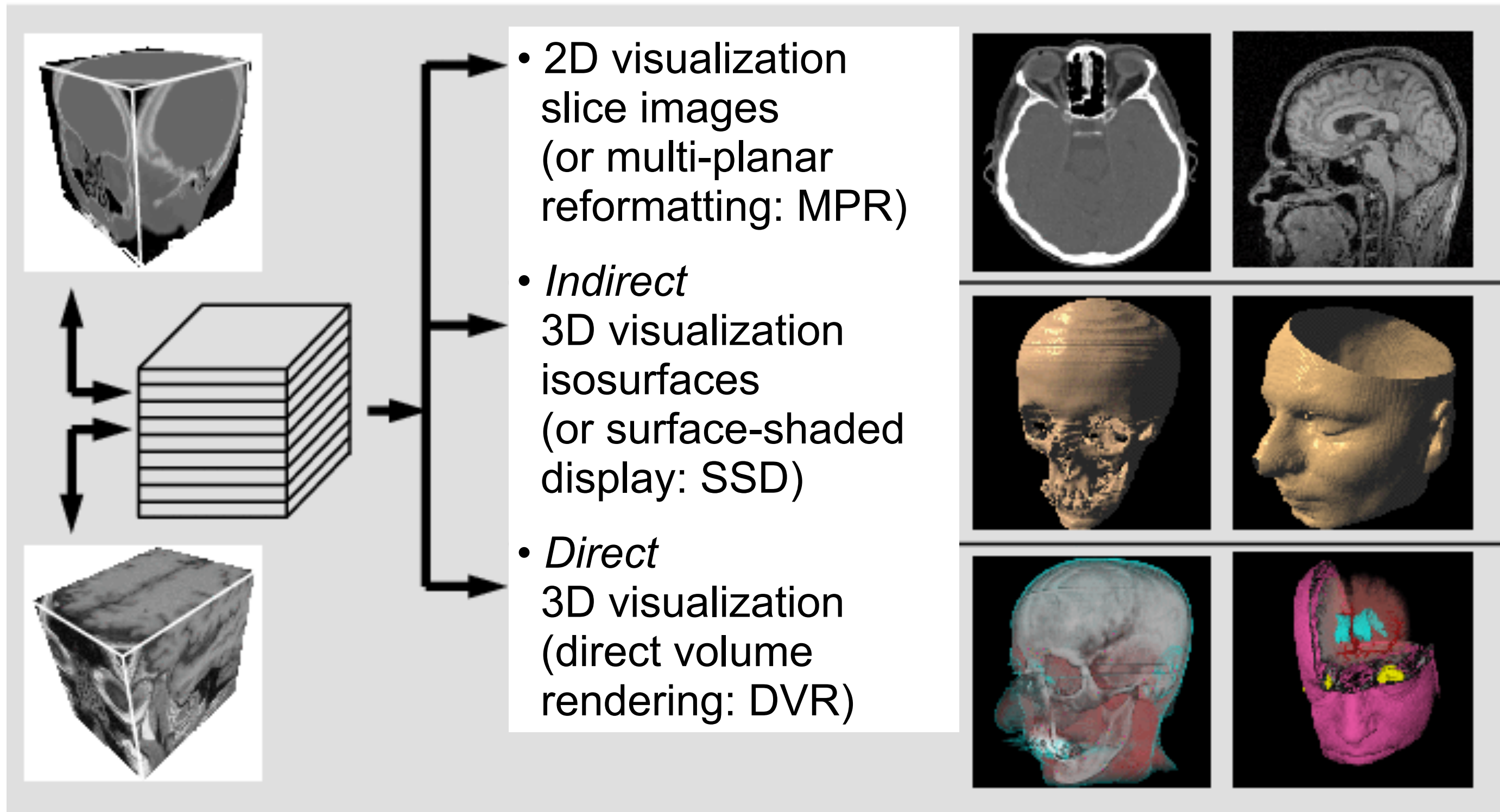
- Techniques for 2D scalar fields
 - Transform 3D data set to 2D
 - Then apply 2D methods
- Indirect volume rendering techniques (e.g. surface fitting)
 - Convert/reduce volume data to an intermediate representation (surface representation), which can be rendered with traditional techniques
- Direct volume rendering
 - Consider the data as a semi-transparent gel with physical properties and directly get a 3D representation of it

Volume Visualization Approaches

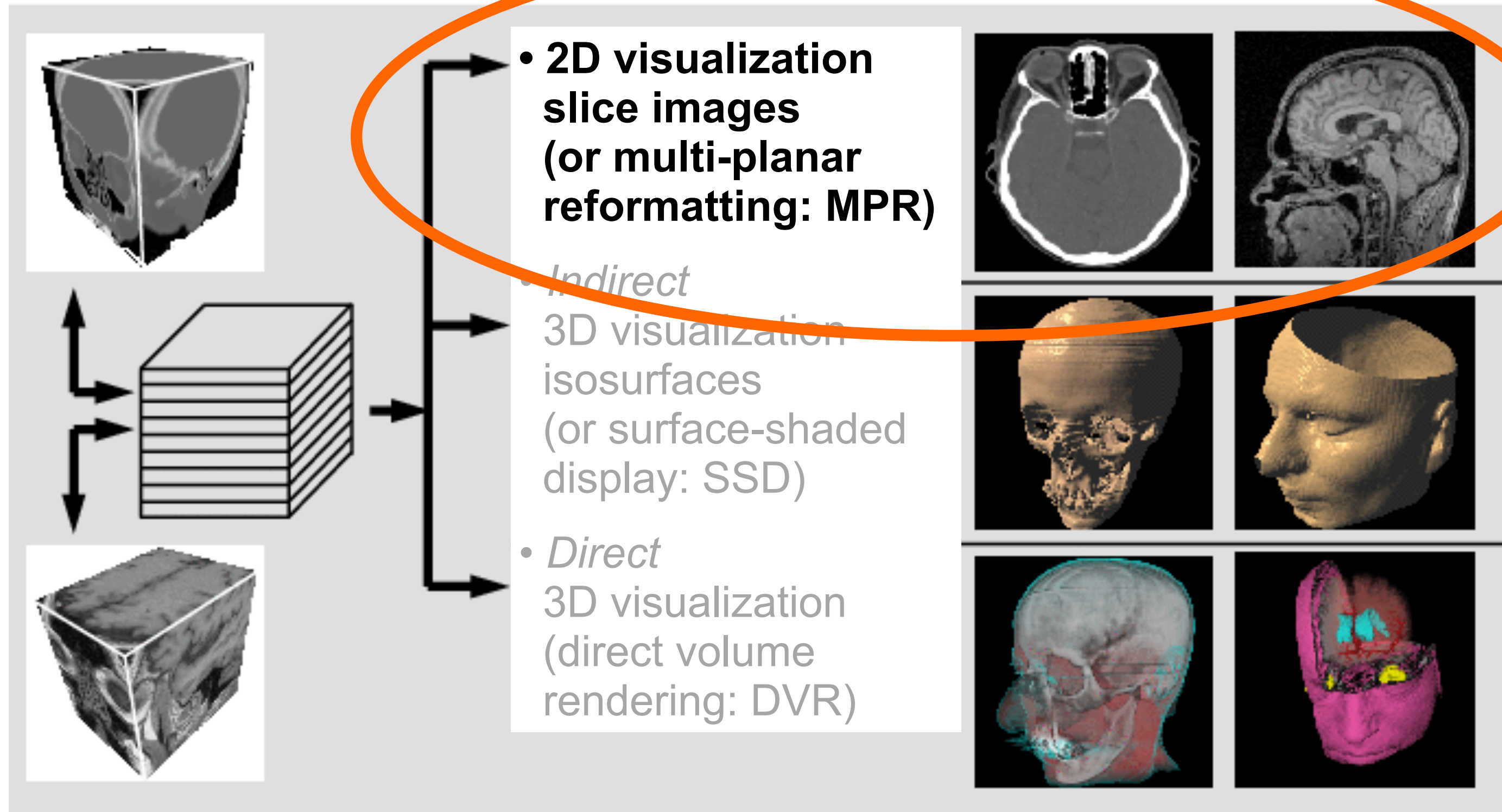
- **Slicing:**
Display the volume data, mapped to colors, on a slice plane
- **Isosurfacing:**
Generate opaque/semi-transparent surfaces
- **Transparency effects:**
Volume material attenuates reflected or emitted light



Volume Visualization Approaches

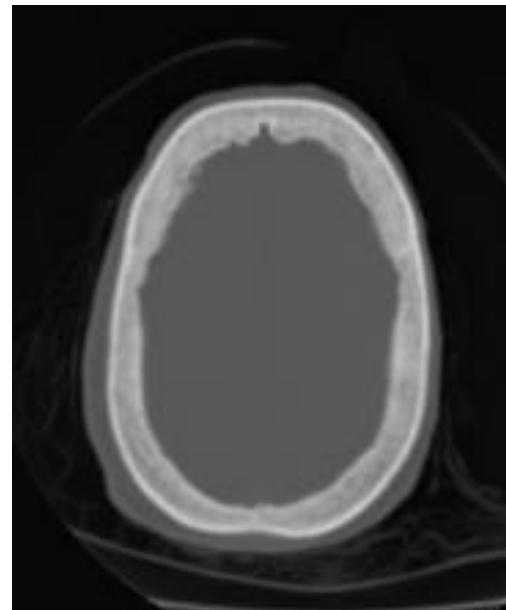


Volume Visualization by Slicing

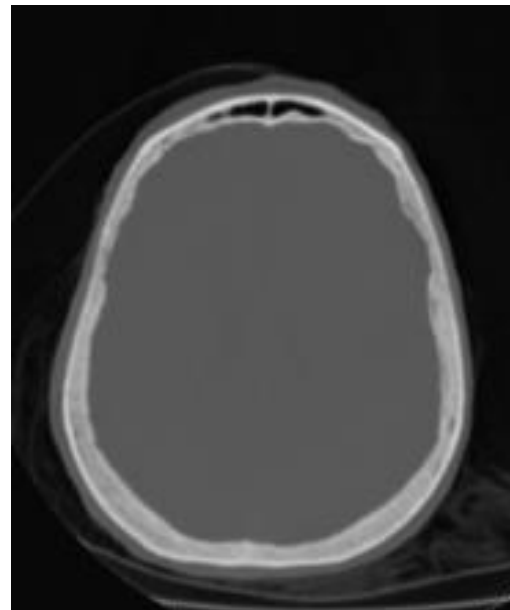


Volume Visualization by Slicing

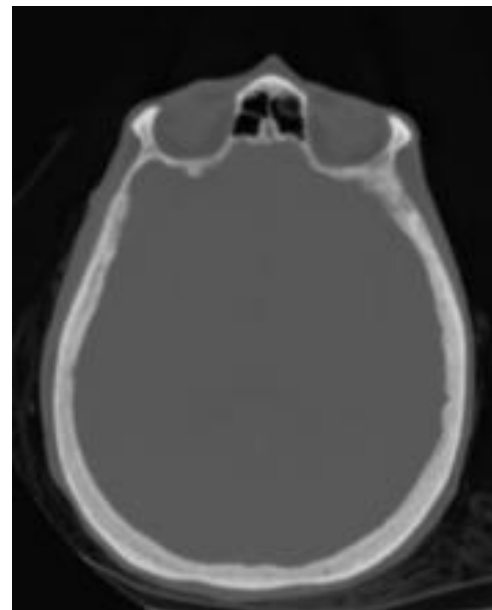
- 2D approach: Orthogonal slicing
 - Interactively resample the data on slices perpendicular to the x-, y-, z-axis
 - Use visualization techniques for 2D scalar fields
 - Color coding
 - Isolines
 - Height fields



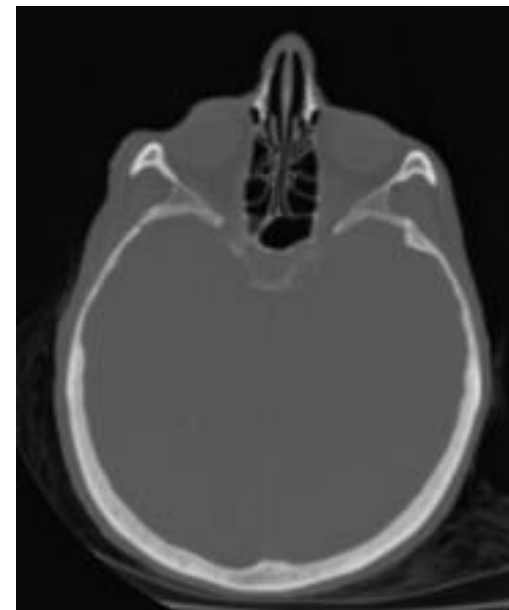
Slice 20



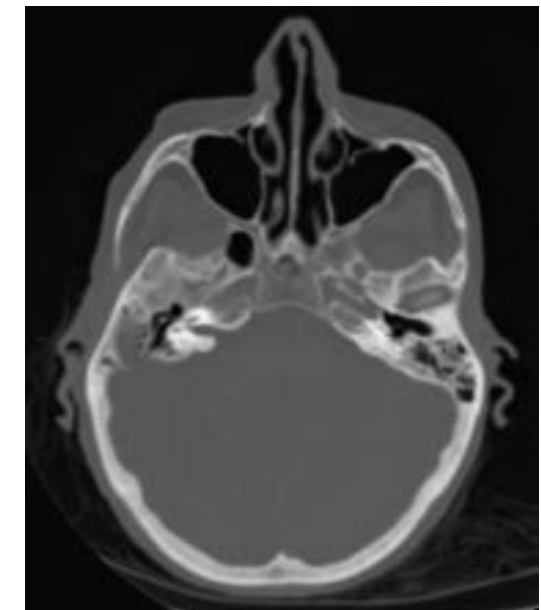
30



40



50



60

CT data set

Volume Visualization by Slicing

- **Alternative: Oblique slicing (MPR multiplanar reformatting)**
 - Resample the data on arbitrarily oriented slices
 - Resampling (interpolation)
 - e.g., exploit 3D texture mapping functionality of OpenGL/Direct3D...
 - ...or compute trilinear interpolation manually

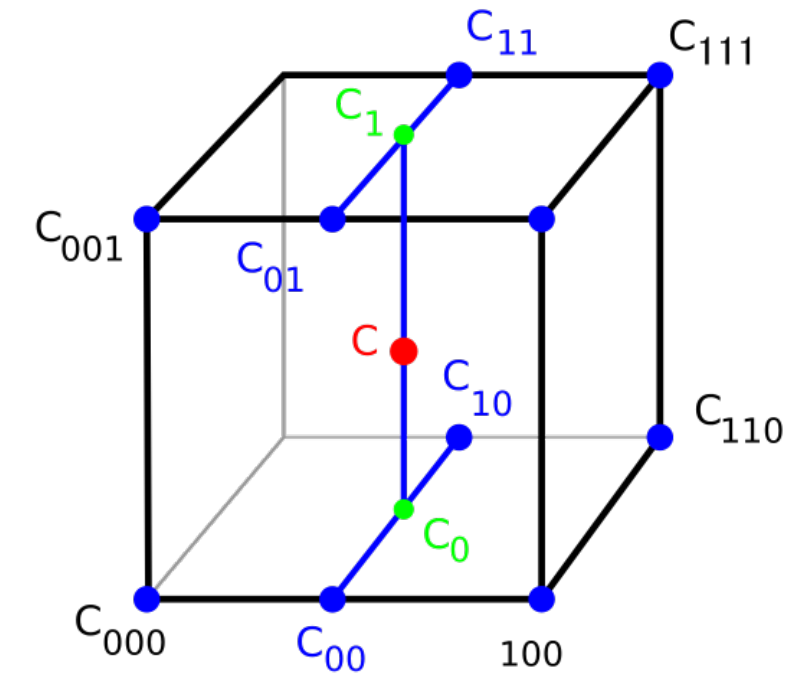
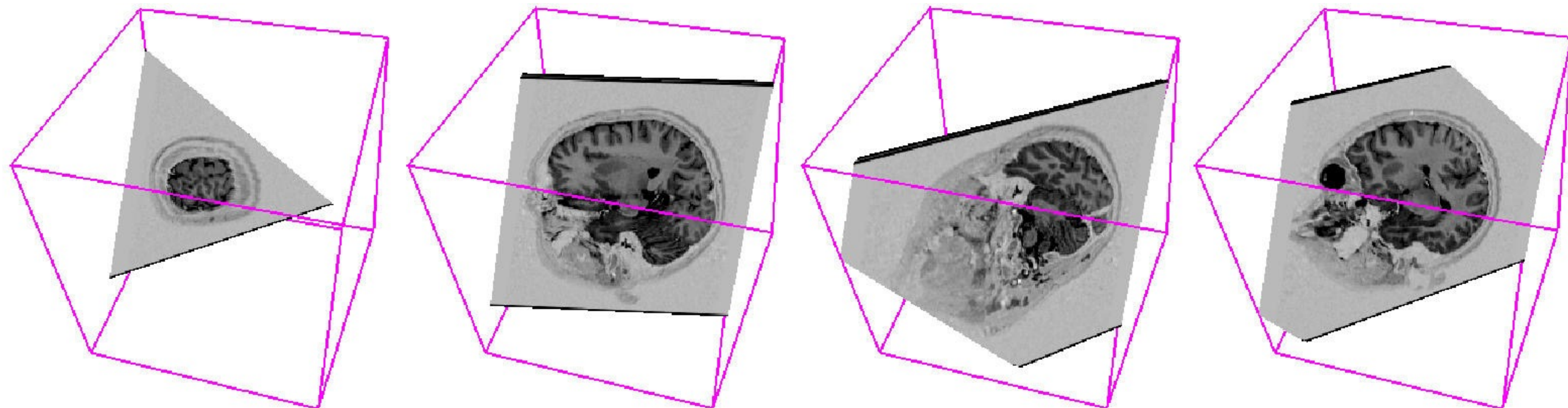
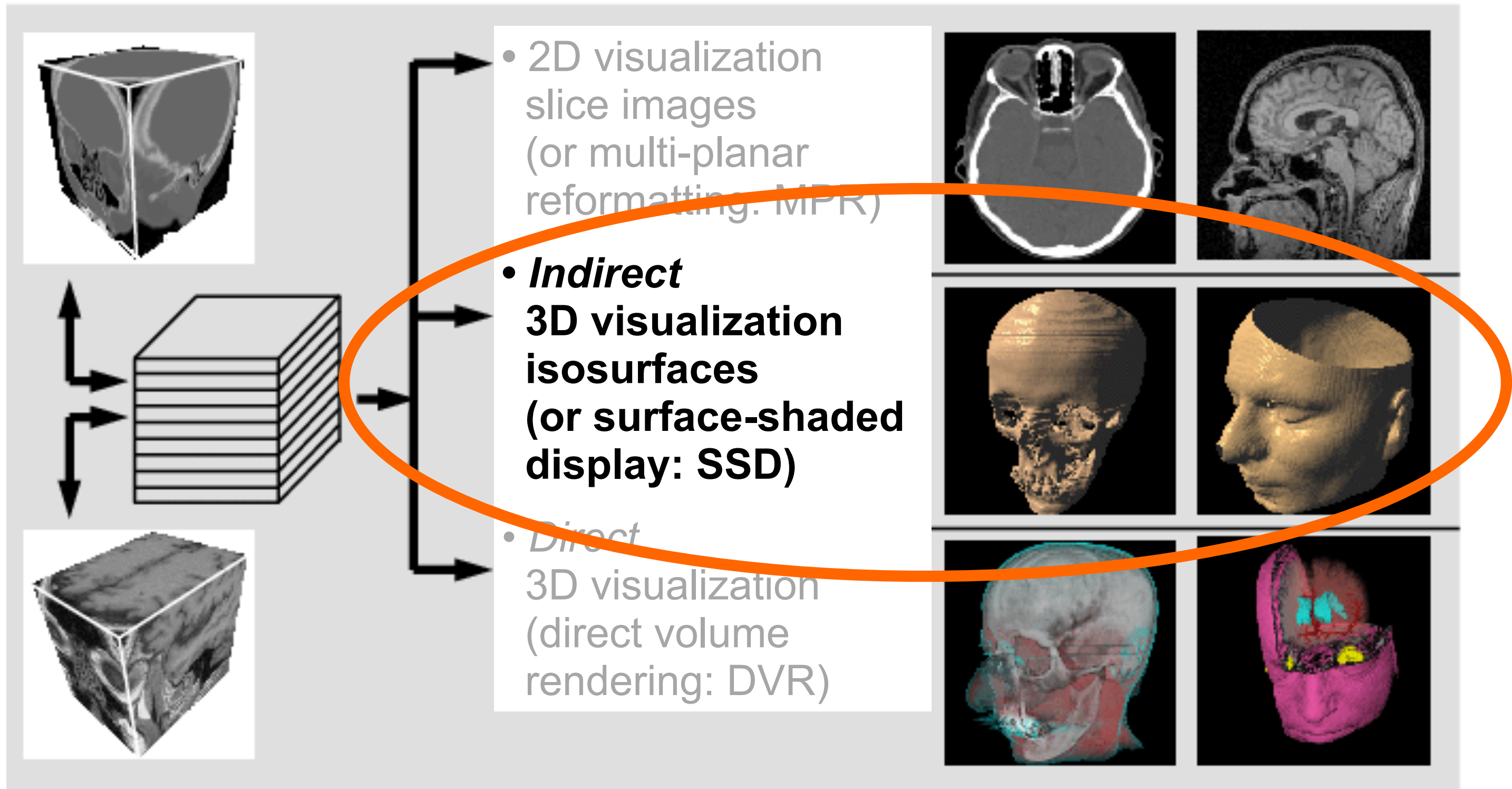


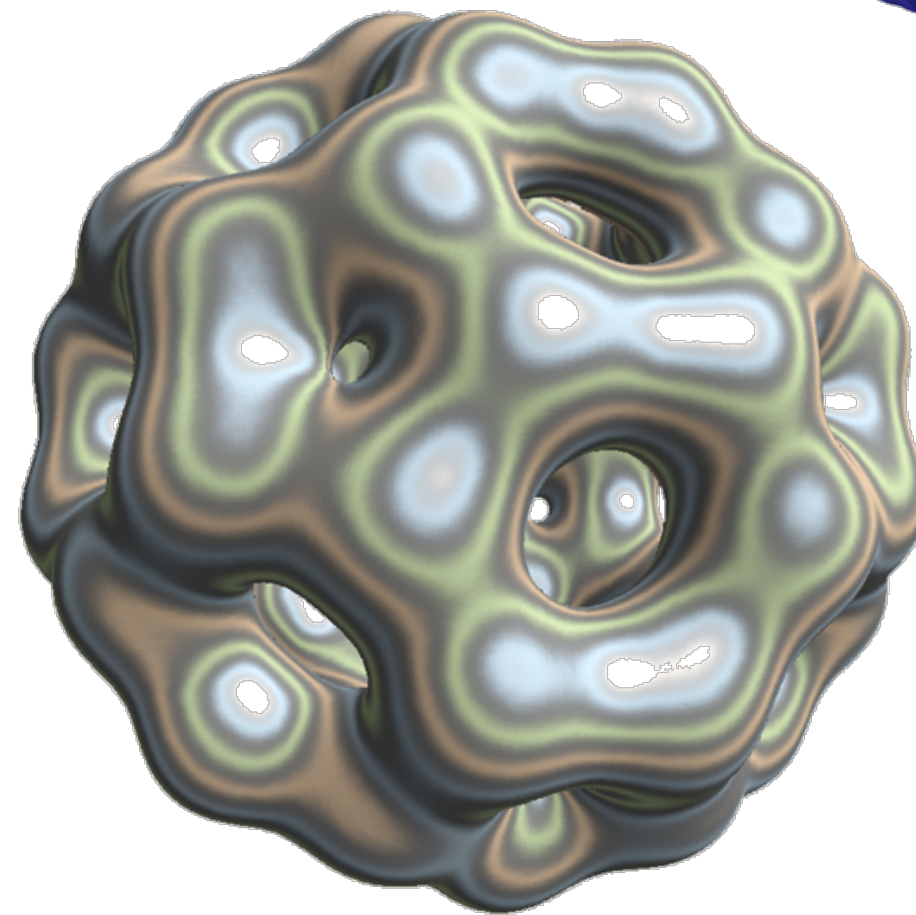
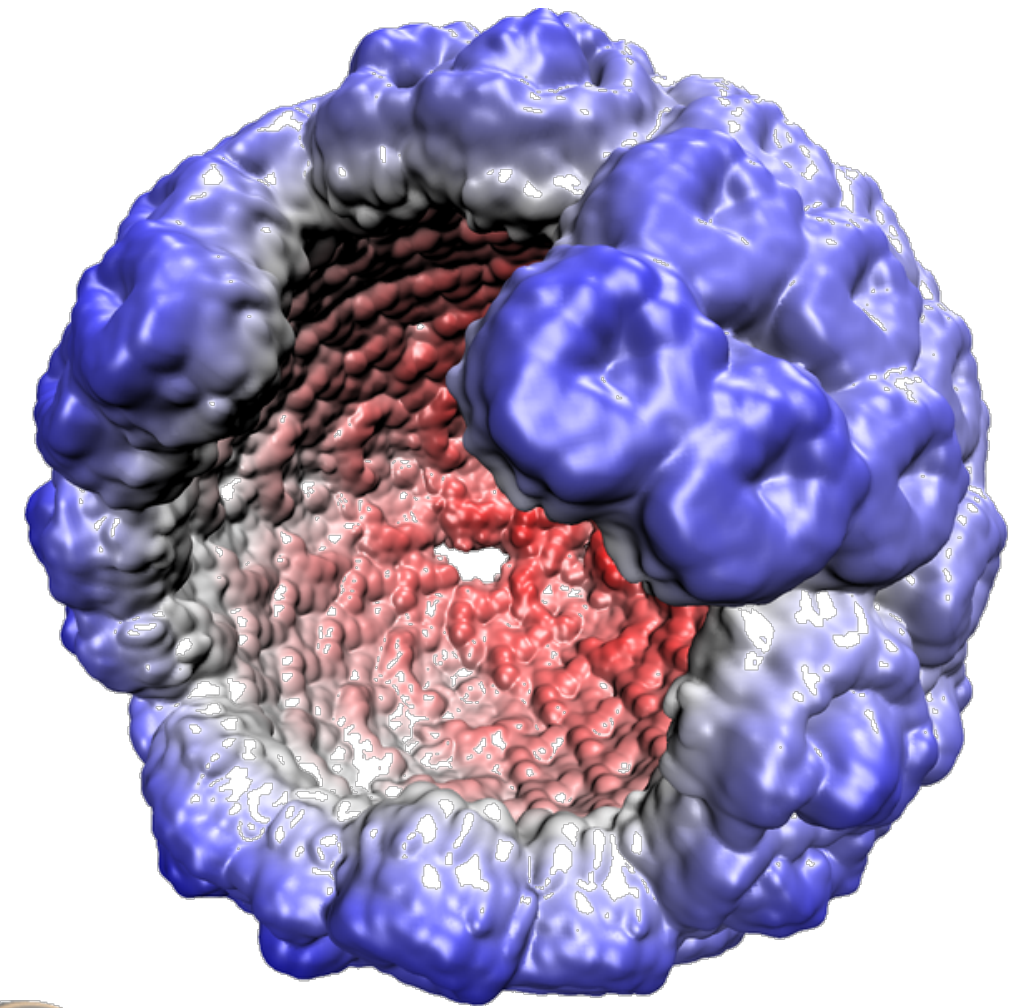
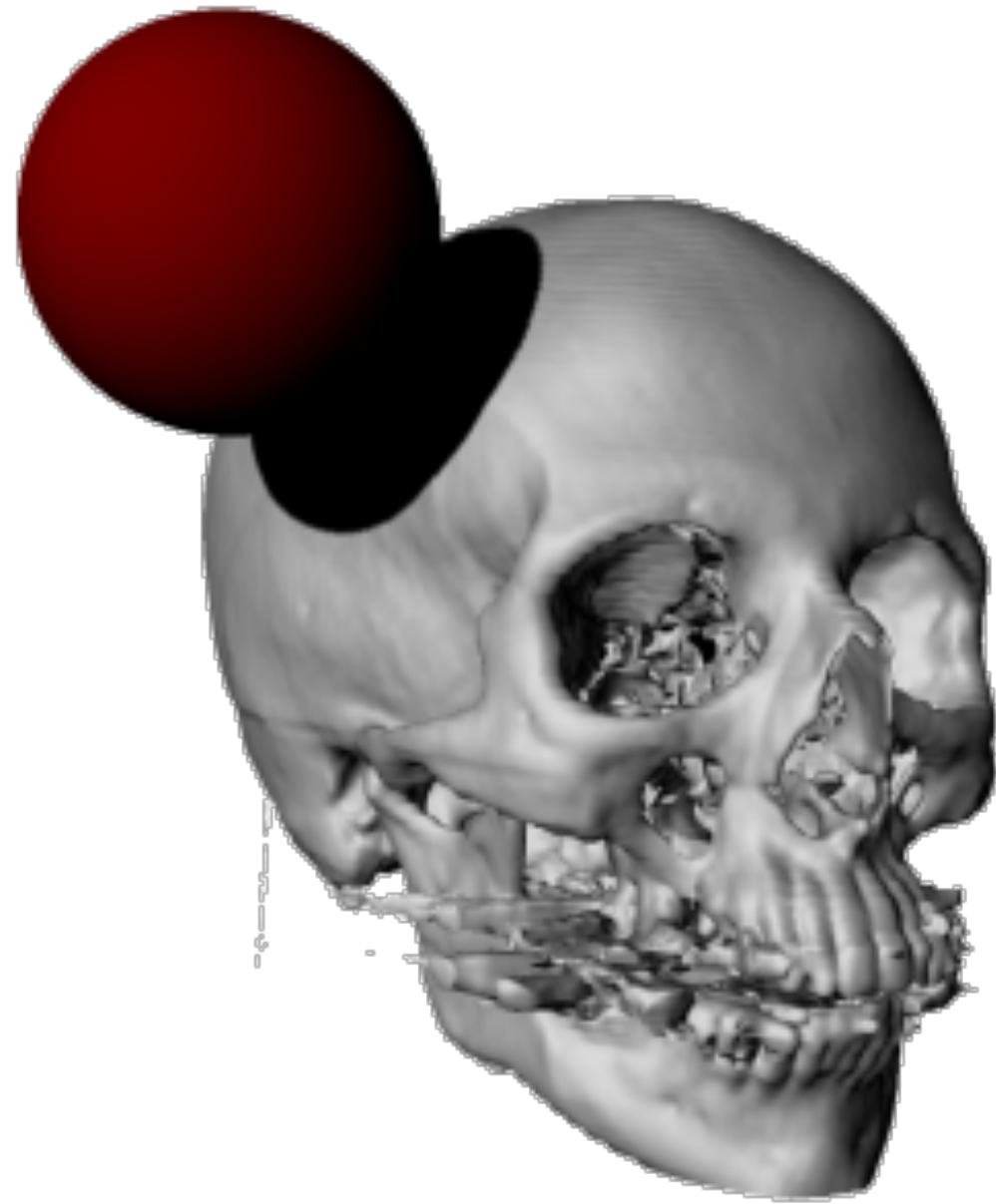
Image source: wikipedia



Volume Visualization by Slicing



Isosurfaces: Examples



Marching Cubes

- **Isosurface extraction by the Marching-Cubes (MC) algorithm**
[Lorensen, Cline 1987] → one of the most cited papers in the CG field (>12,000)
 - Works on the original data
 - Approximates the surface by a triangle mesh
 - Surface is found by linear interpolation along cell edges
 - ***THE* standard geometry-based isosurface extraction algorithm**
 - Extension of Marching Squares to 3D
 - Assumes a uniform or rectilinear grid
- Similar for general hexahedral grids
- Related Marching algorithms for unstructured grids

Marching Cubes: Algorithm

- The core Marching-Cubes algorithm
 - Cell (cube) consists of 8 grid values:
($i+[01]$, $j+[01]$, $k+[01]$)
 1. Consider a cell
 2. Classify each vertex as inside or outside
 3. Build an index
 4. Get edge list from table[index]
 5. Interpolate the edge location
 6. Compute gradients
 7. Consider ambiguous cases
 7. Go to next cell

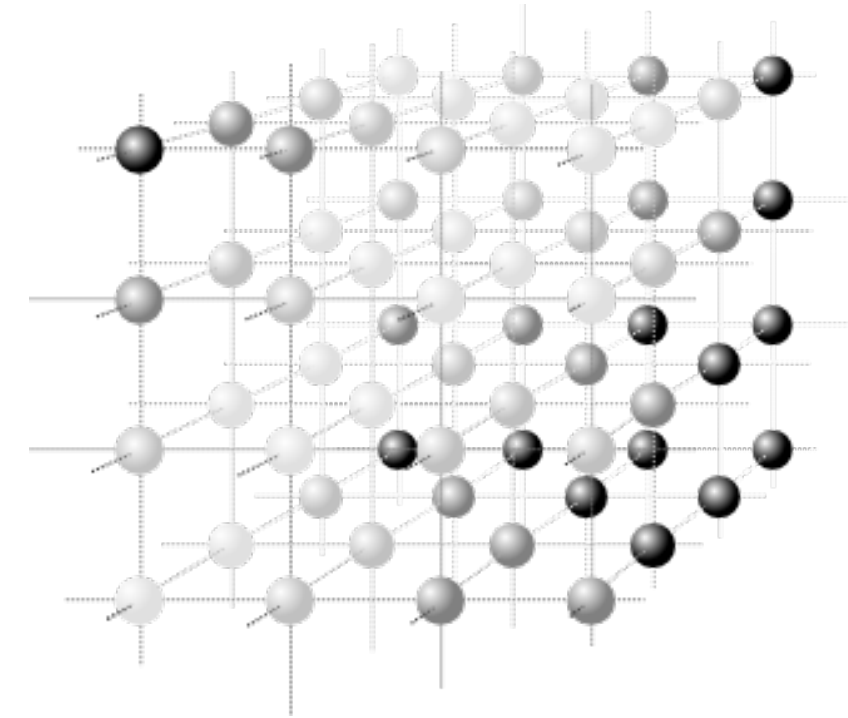
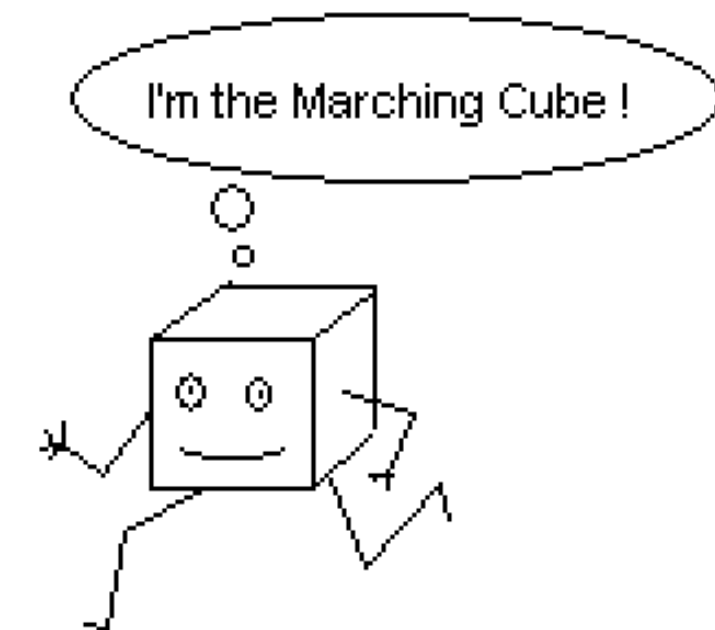
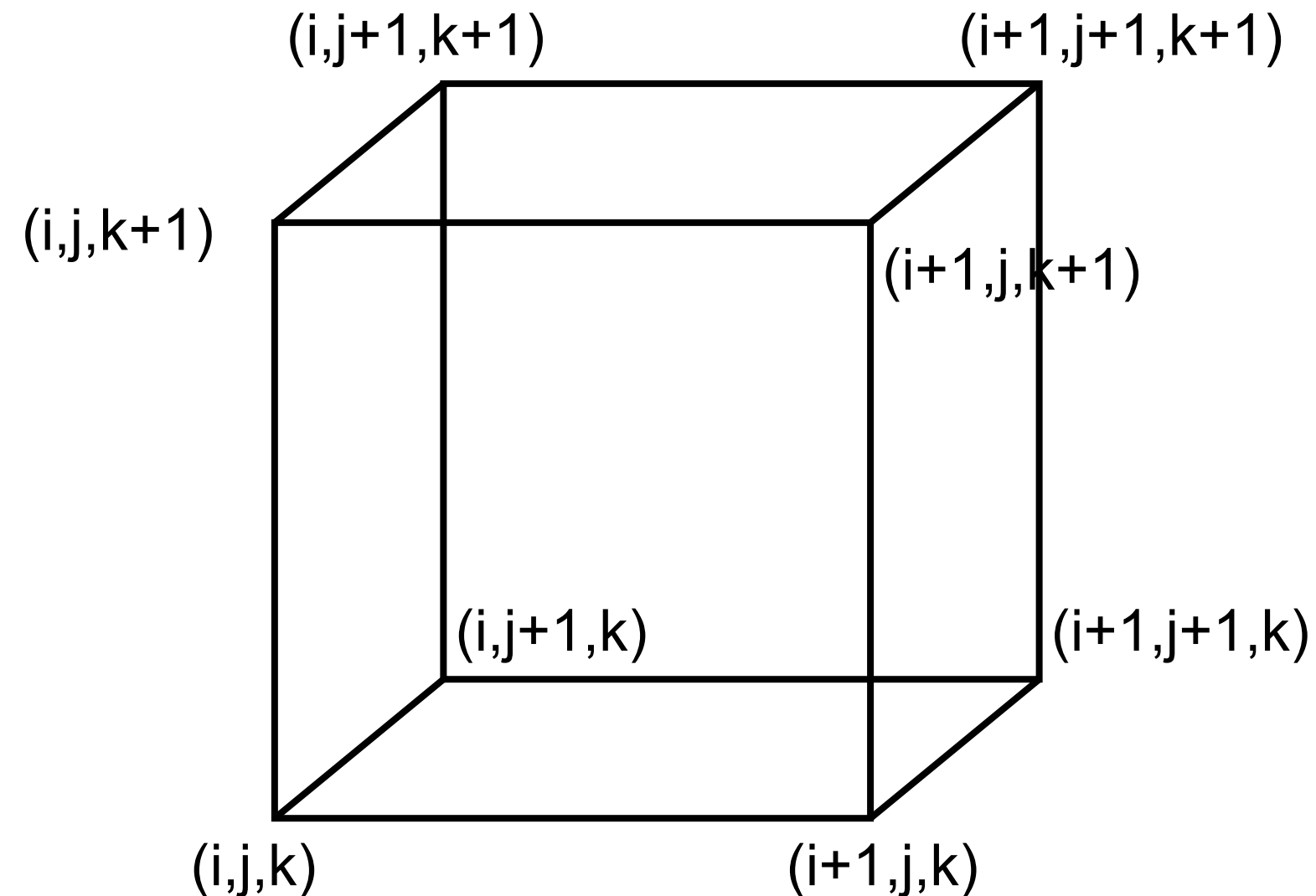


Image source: wikipedia



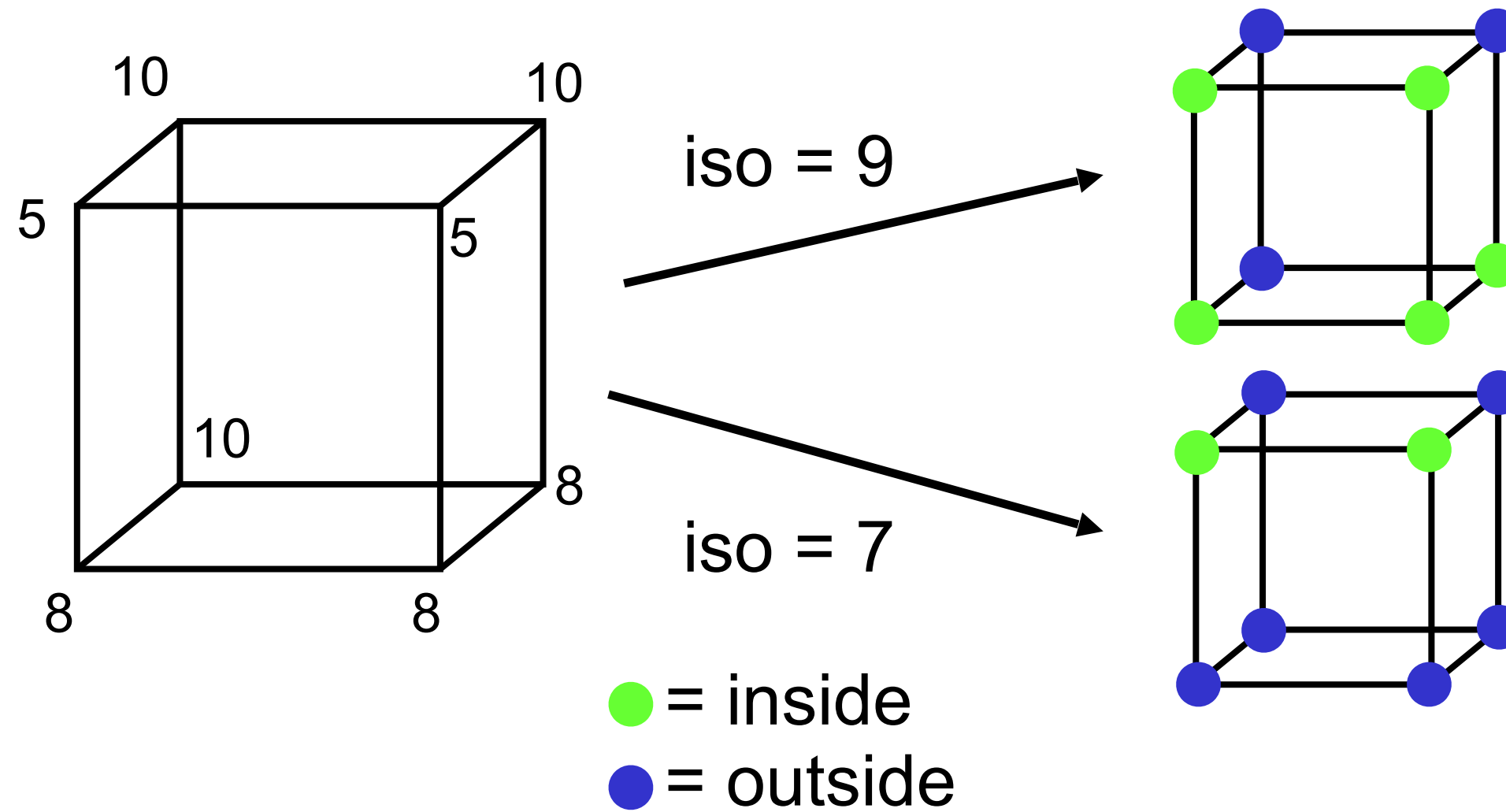
Marching Cubes: Algorithm

- Step 1: Consider a cell defined by eight data values



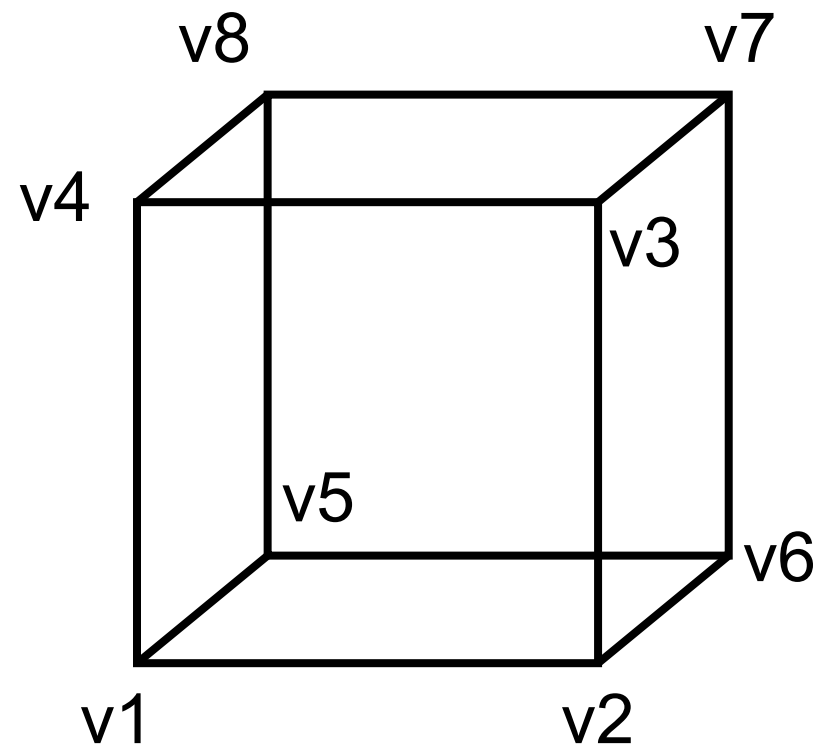
Marching Cubes: Algorithm

- Step 2: Classify each cell according to whether it lies
 - Outside the surface (value > isosurface value)
 - Inside the surface (value <= isosurface value)

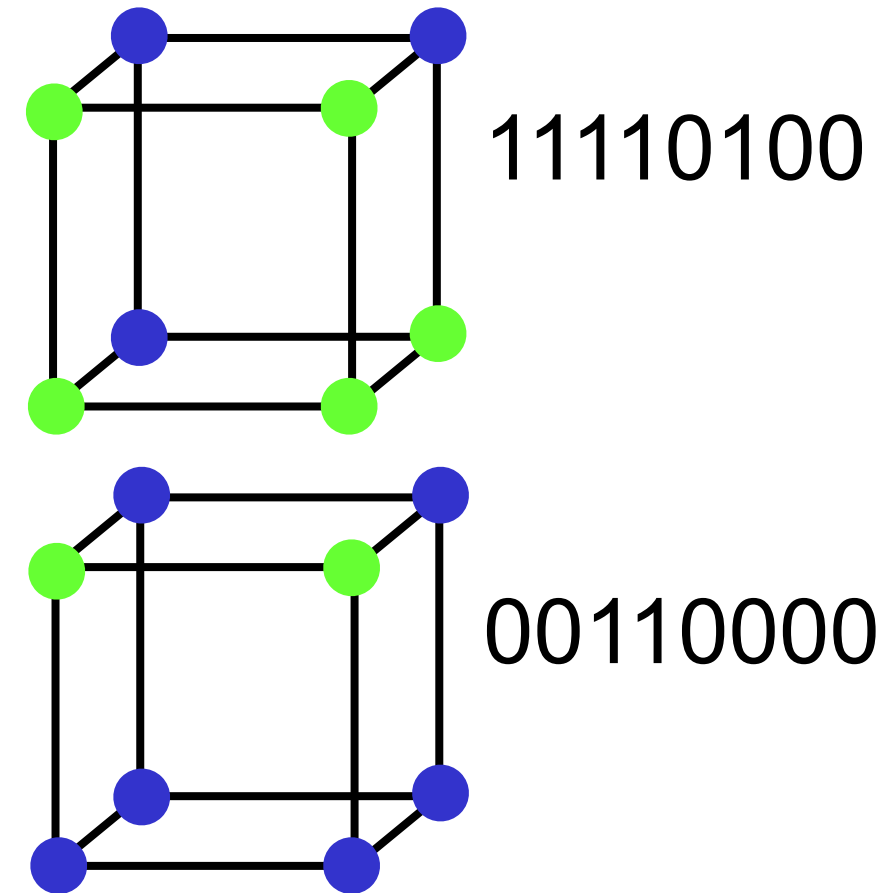


Marching Cubes: Algorithm

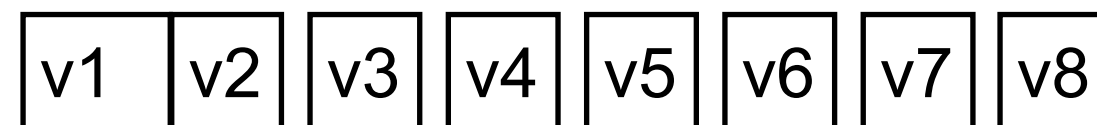
- Step 3: Use the binary labeling of each cell to create an index



● inside = 1
● outside = 0



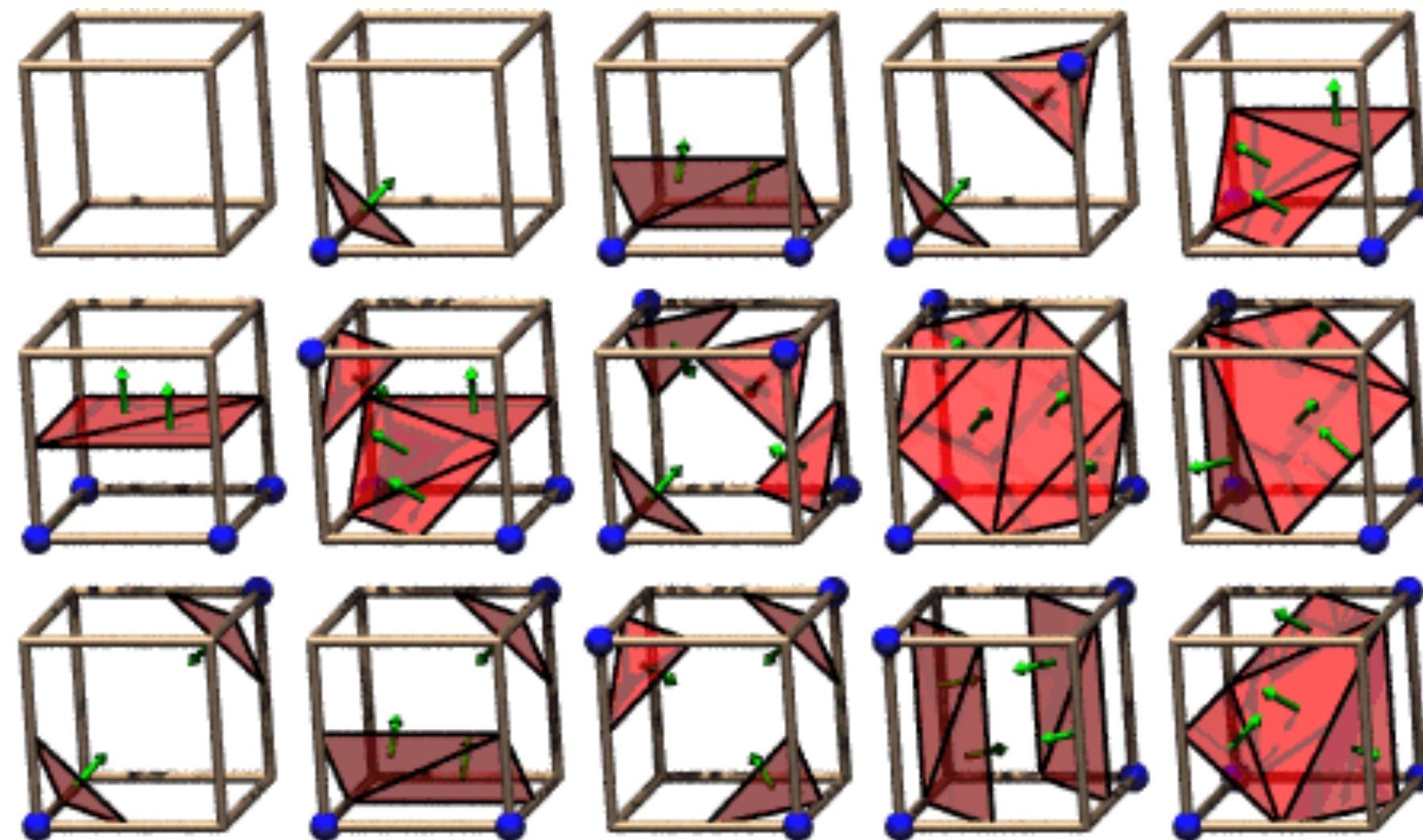
Index:



Binary index number = integer

Marching Cubes: Algorithm

- Step 4: For a given index, access an array storing a list of edges
 - All 256 cases can be derived from $1 + 14 = 15$ base cases due to symmetries
 - Each case creates at most 5 triangles (dual cases for inverted signs)



The 15 Cube Combinations

Marching Cubes: Algorithm

- Step 4 *cont.*: Get edge list from table

- Example:

Index = 10110001

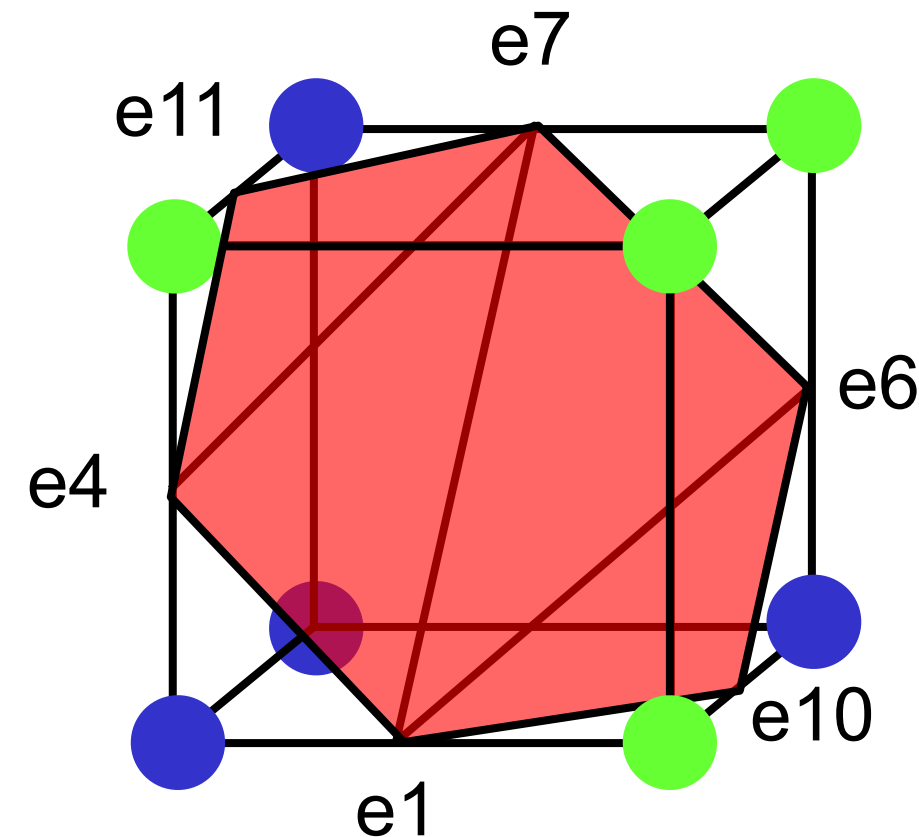
triangle 1 = e4, e7, e11

triangle 2 = e1, e7, e4

triangle 3 = e1, e6, e7

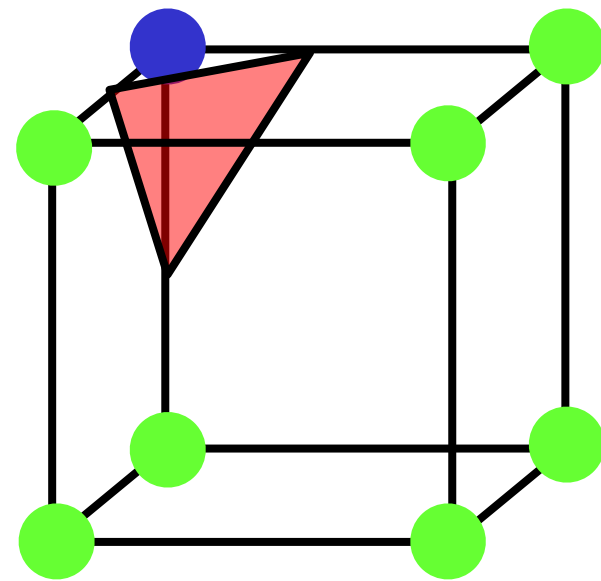
triangle 4 = e1, e10, e6

- Face normals encoded implicitly by order of vertices
- Normal points to higher (or lower) values of the field \rightarrow inside/outside
- Vertex normals can be computed by averaging all surrounding face normals



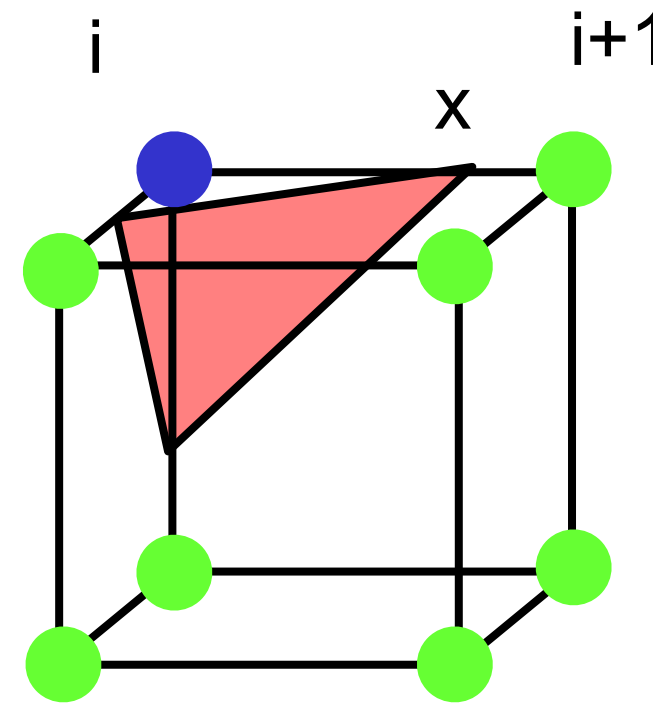
Marching Cubes: Algorithm

- Step 5: For each triangle edge, find the vertex location along the edge using linear interpolation



$c = 3$

● = 10
● = 0



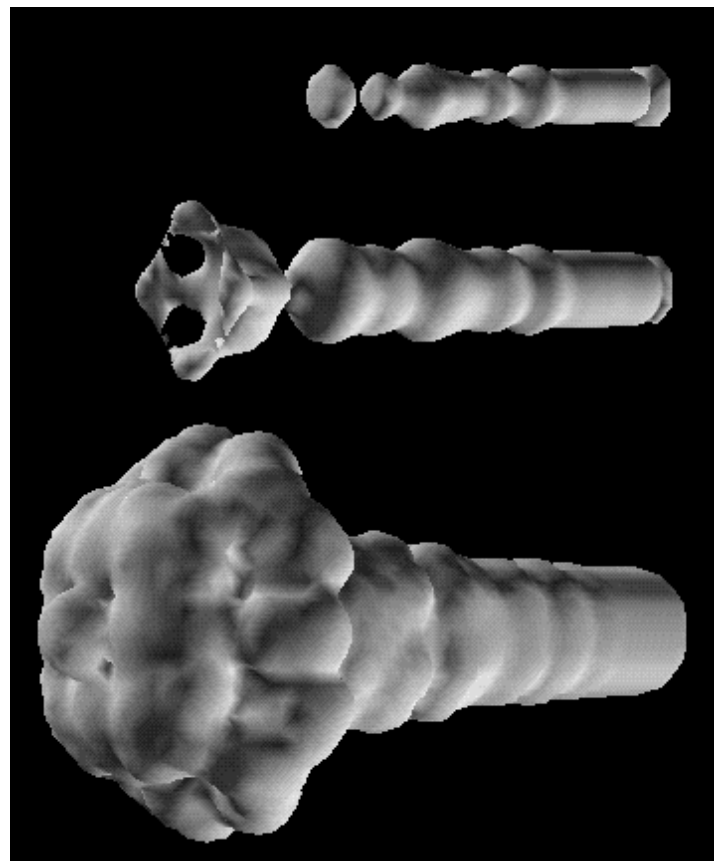
$c = 8$

if all edge lengths = 1 $x = i + (c - v[i]) / (v[i+1] - v[i])$

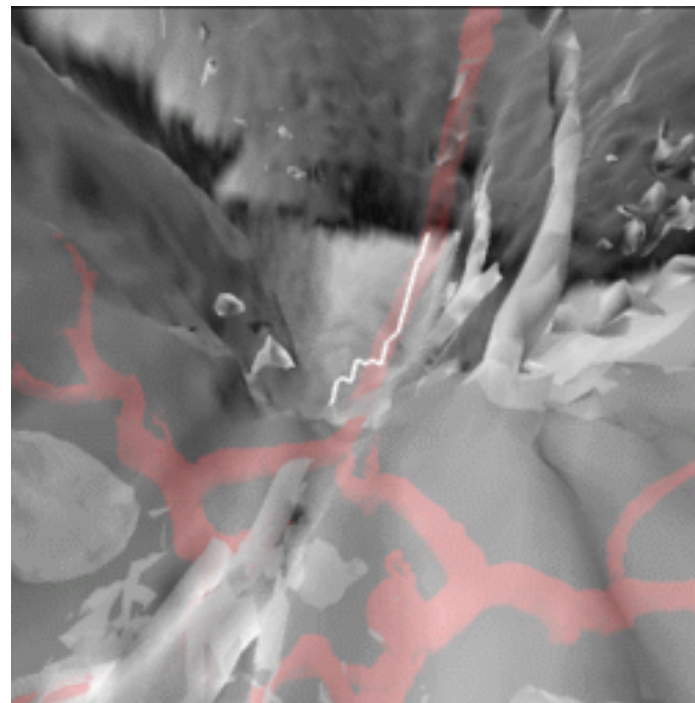
otherwise $x = [(v[i+1] - c)x[i] + (c - v[i])x[i+1]] / (v[i+1] - v[i])$

Marching Cubes: Examples

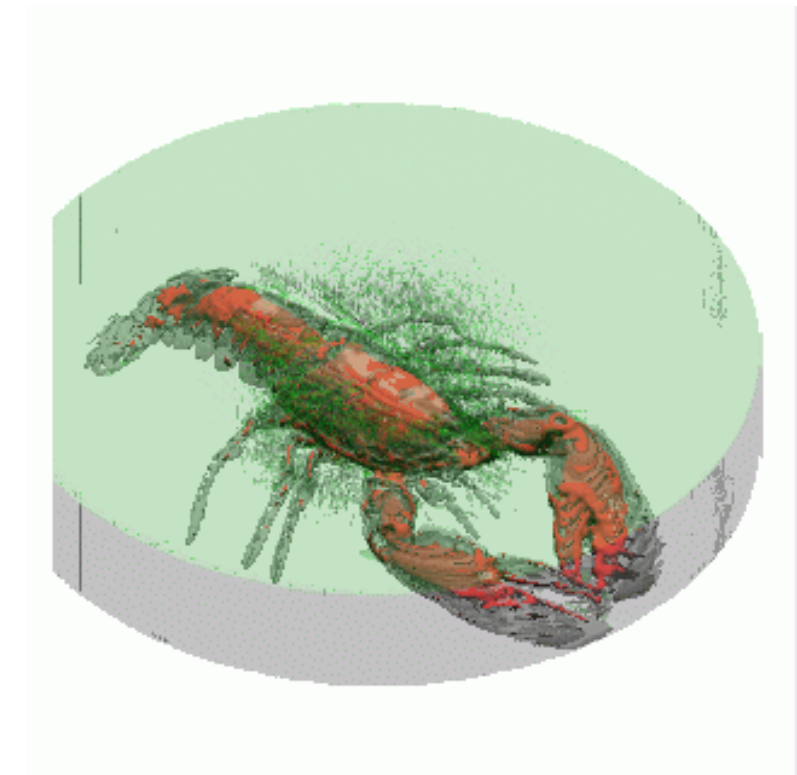
- Possibly overlay of several isosurfaces



1 Isosurface



2 Isosurfaces



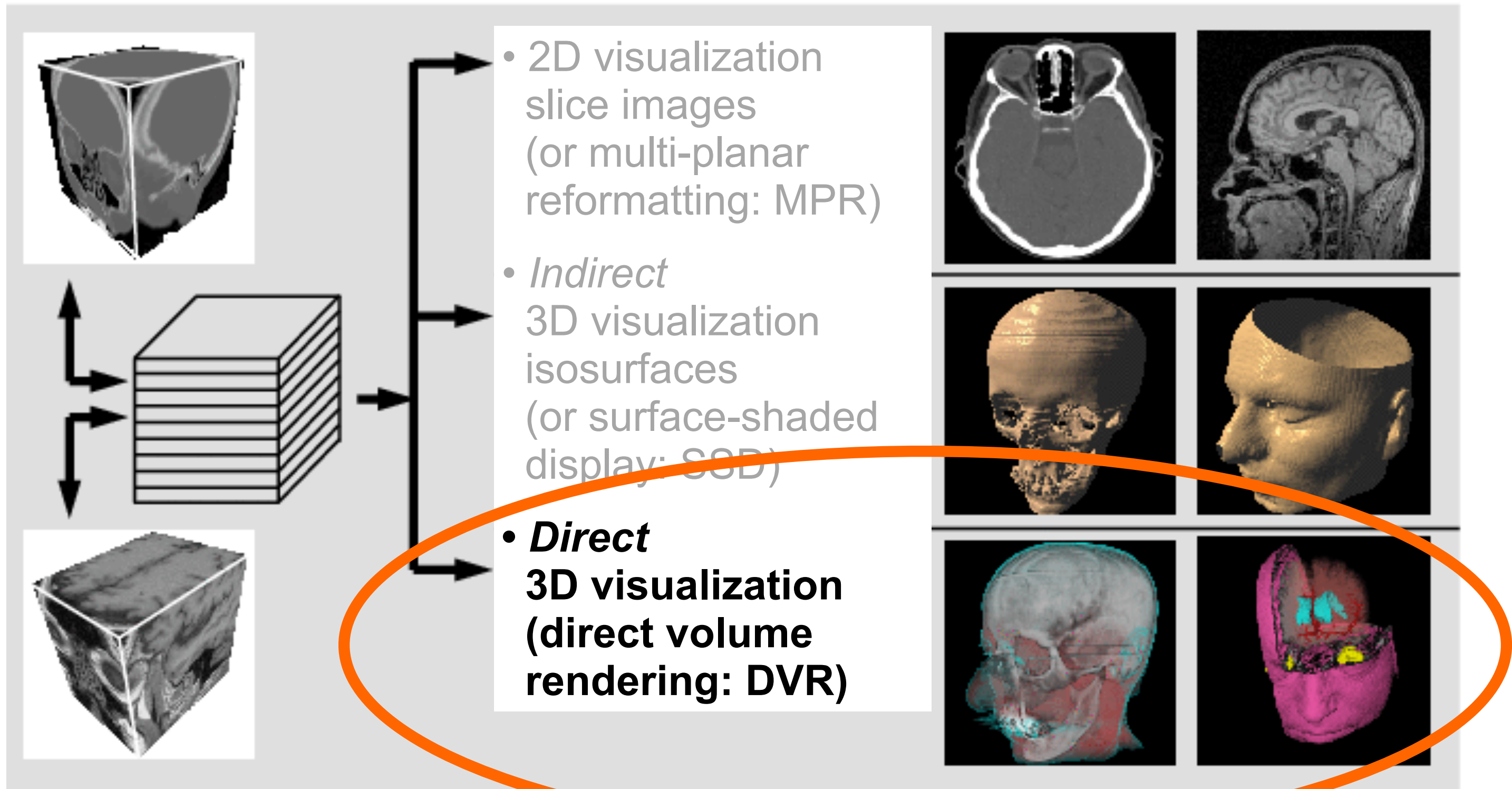
3 Isosurfaces

Marching Cubes: Examples

- Nvidia Cascades Demo for Geforce 8800 (~2008)
 - Real-time generation of volumetric terrain on GPU
 - Marching Cubes isosurface extraction in Geometry Shader

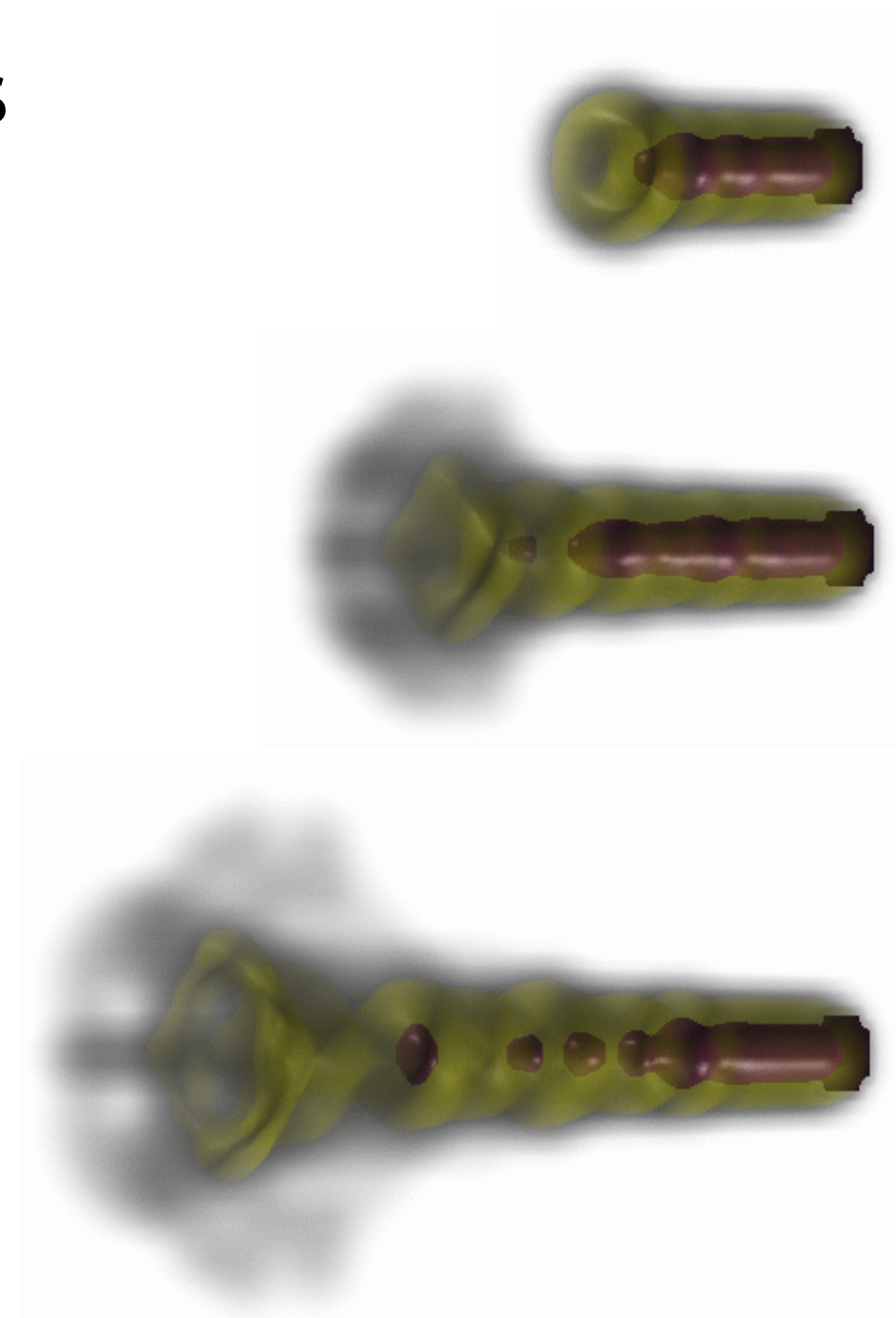


Volume Visualization by Slicing



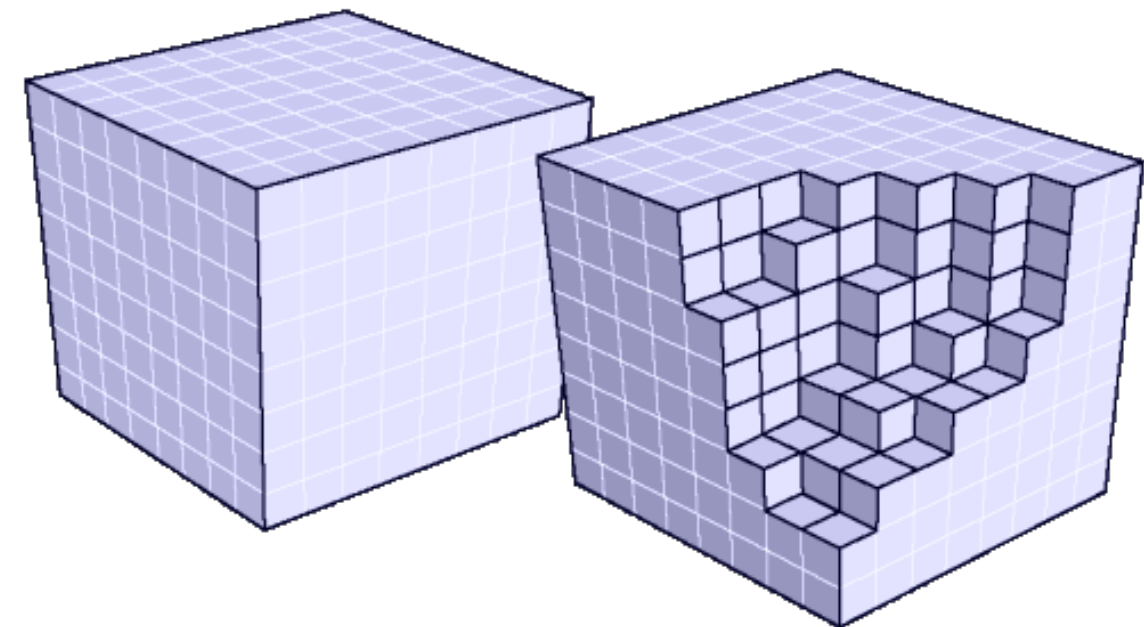
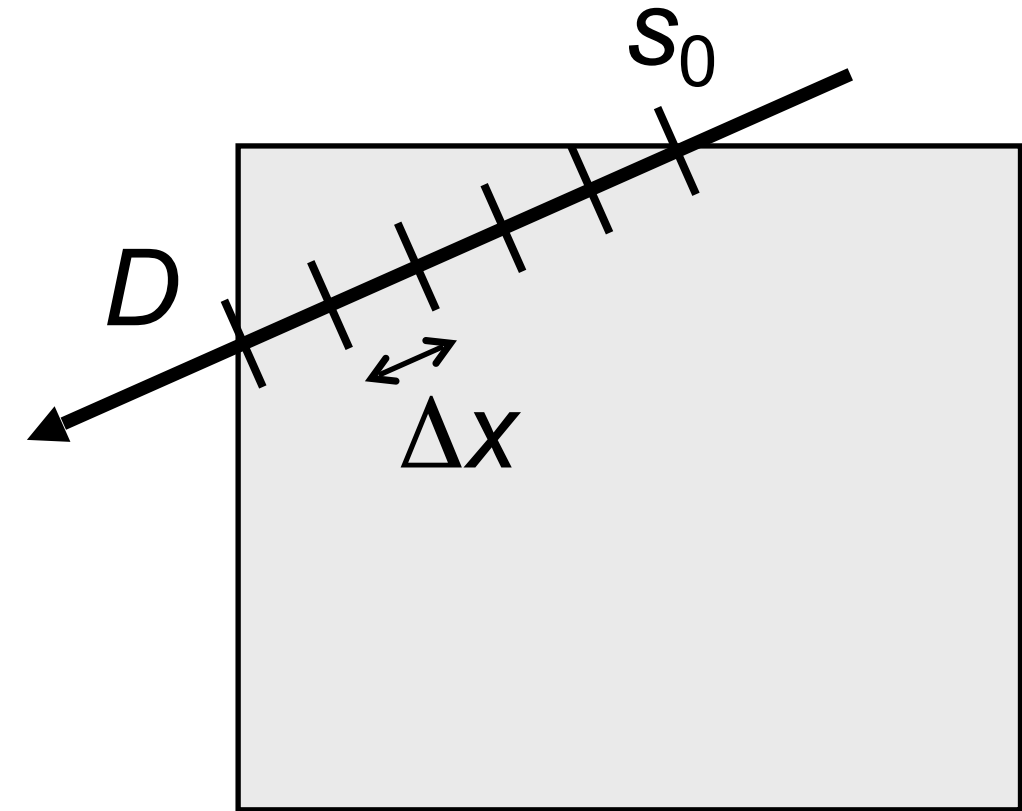
Direct Volume Rendering

- Directly get a 3D representation of the volume data
 - The data is considered to represent a semi-transparent light-emitting medium
 - Approaches are based on the laws of physics (emission, absorption, scattering)
 - The volume data is used as a whole (look inside, see all interior structures)



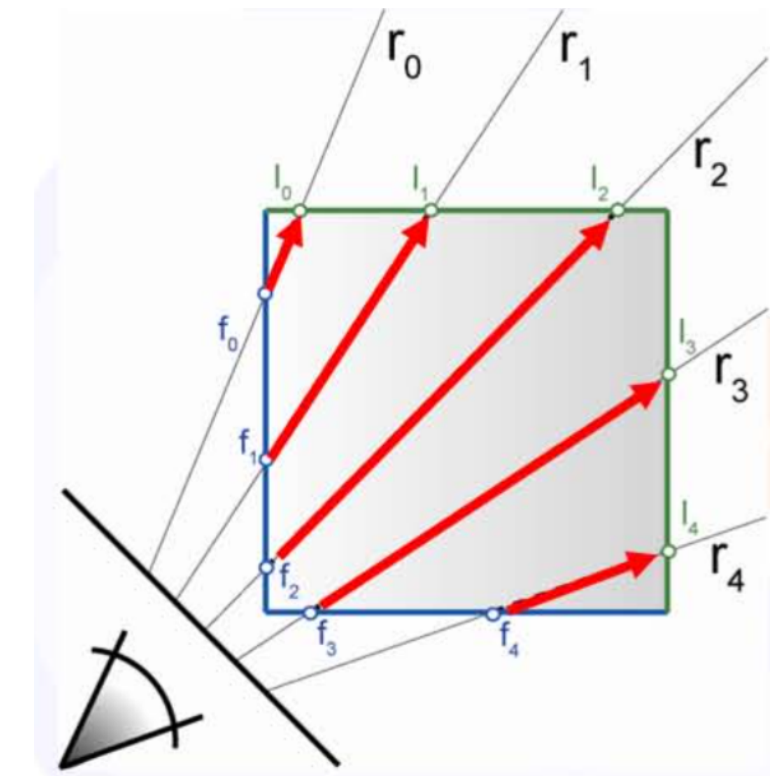
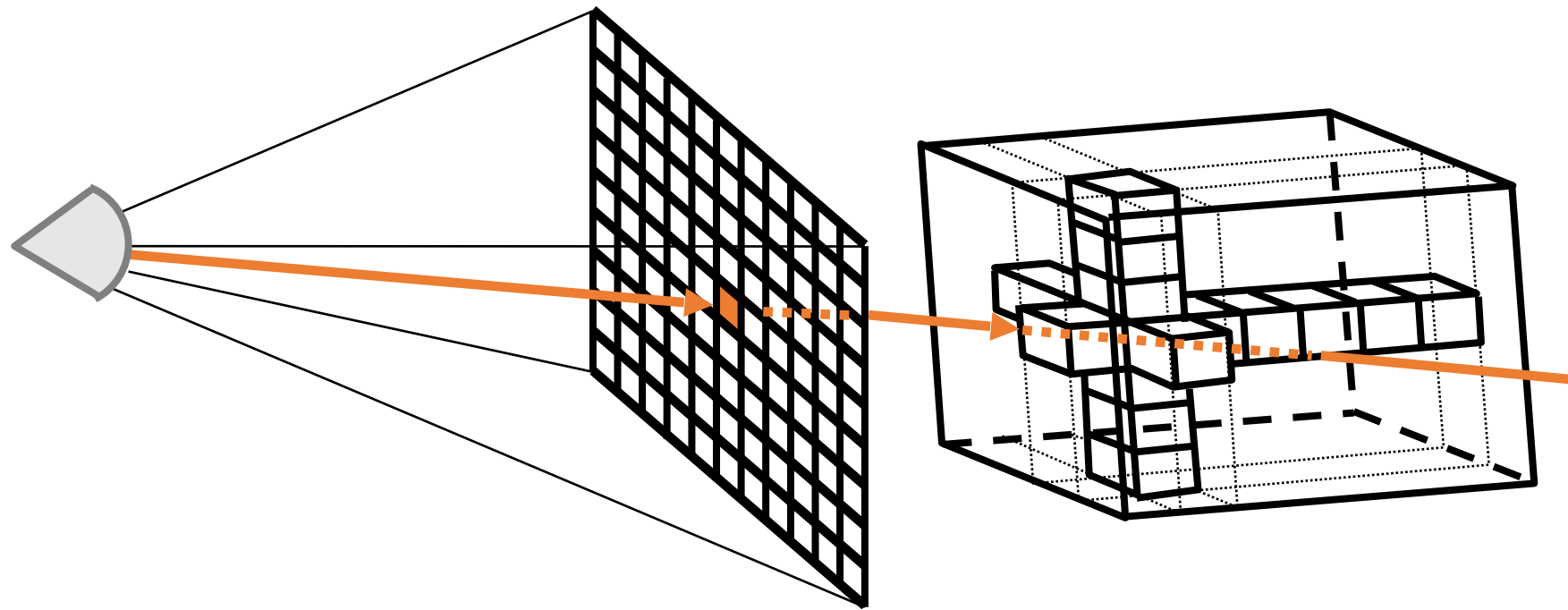
Direct Volume Rendering

- Optical model:
volume rendering integral
- Integral approximated by sum
along virtual light rays
- Compositing (accumulation of
color and opacity) depends on
 - Incoming light (RGB)
 - Opacity ($A = \alpha = 1 - \text{transparency}$)



Ray Casting

- Similar to ray tracing in surface-based computer graphics
- In volume rendering we usually only deal with primary rays; hence: *ray casting* (or *ray marching*)
- Natural **image-order** technique
- Performed pixel-by-pixel



Compositing along Rays

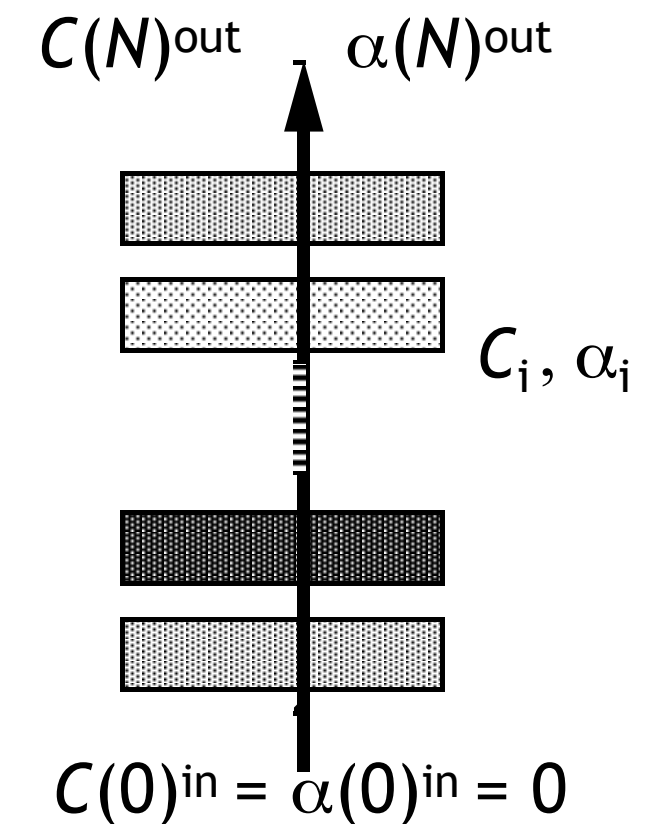
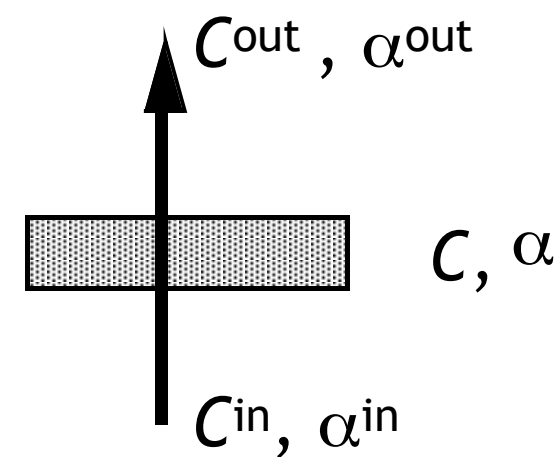
- Front-to-back compositing
 - Combine (sorted) color and opacity values along a ray
 - Most often used in ray casting
 - Allows for early ray termination
- Compositing equation:

$$C^{\text{out}} = C^{\text{in}} + (1 - \alpha^{\text{in}}) C \alpha$$

$$\alpha^{\text{out}} = \alpha^{\text{in}} + (1 - \alpha^{\text{in}}) \alpha$$

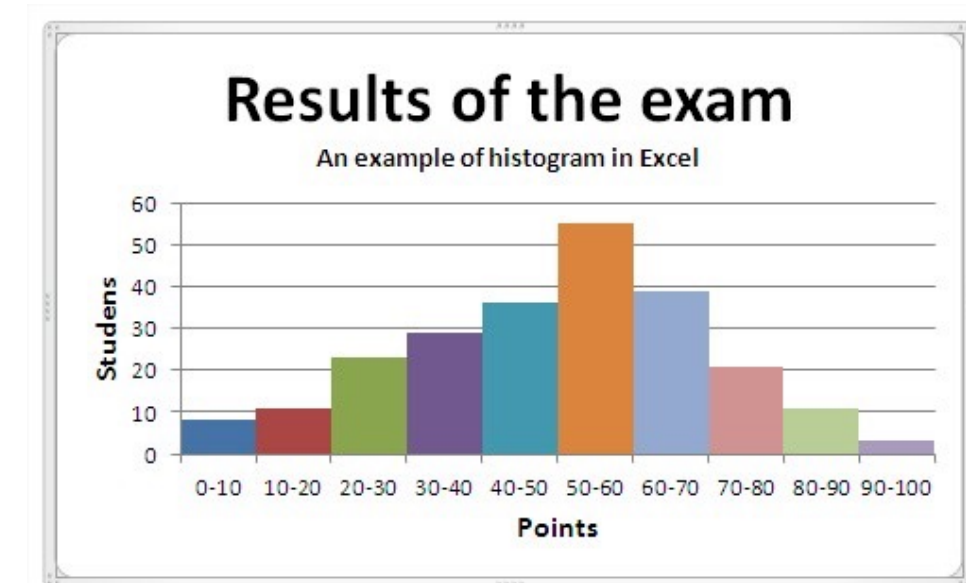
$$C(i)^{\text{in}} = C(i-1)^{\text{out}}$$

$$\alpha(i)^{\text{in}} = \alpha(i-1)^{\text{out}}$$



Problem: Classification

- Missing link so far between
 - Scalar values of the data set and...
 - ...color & opacity (RGBA) for volume rendering
- Goals and issues
 - Empowers user to select “structures”
 - Extract important features of the data set
 - Classification is non trivial
 - Histogram can be a useful hint
- Standard approach: Transfer function
 - Color table for volume visualization
 - Maps raw scalar value into presentable entities: color, intensity, opacity, etc.

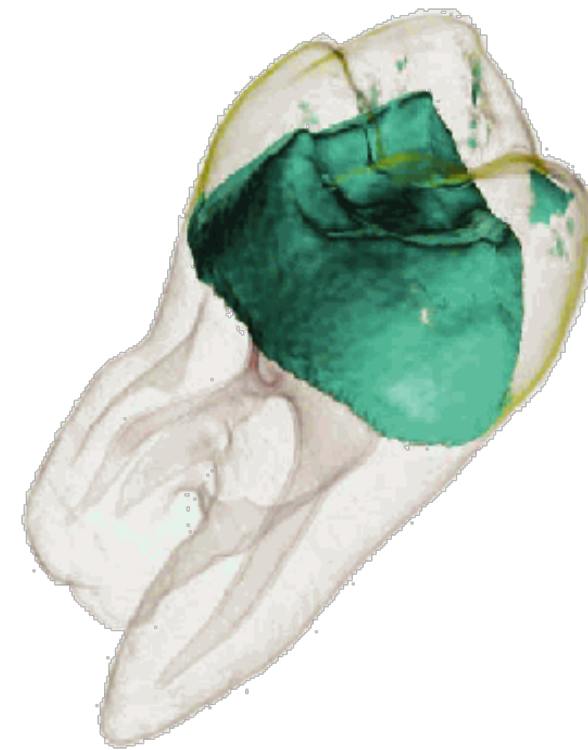
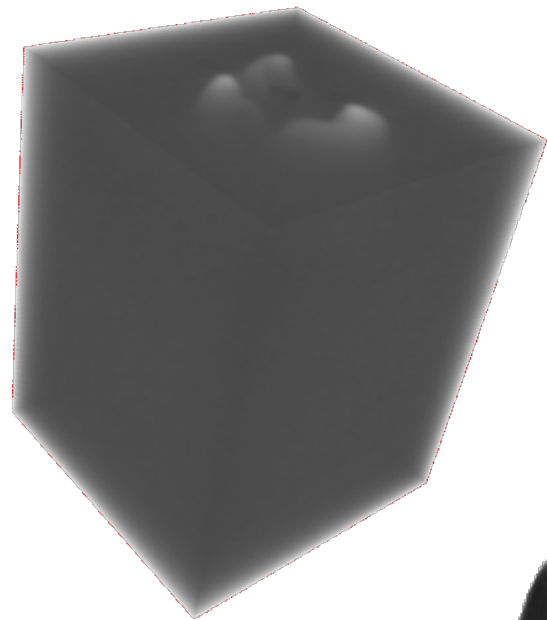


Histogram

Image source: best-excel-tutorial.com

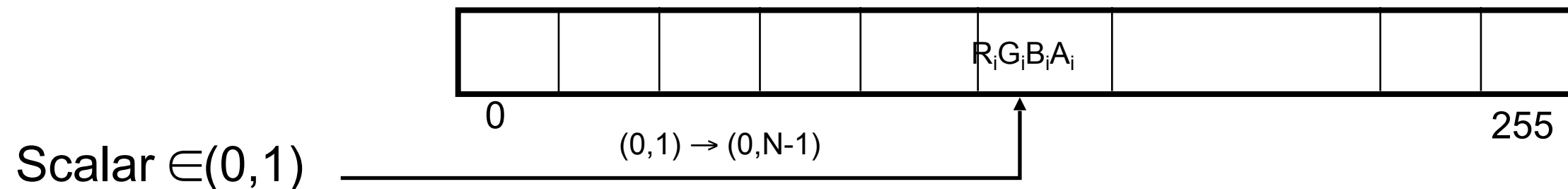
Classification: Transfer Function

- Examples of different transfer functions

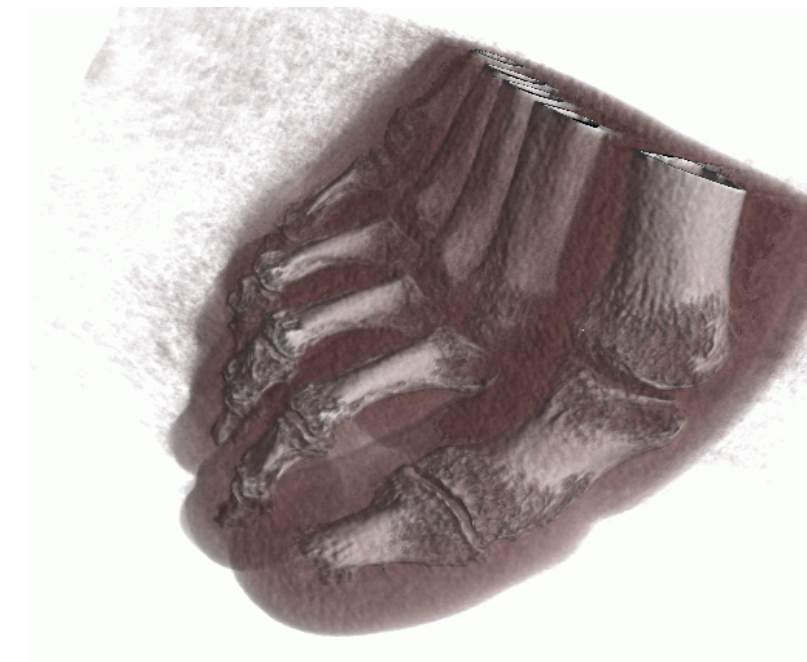
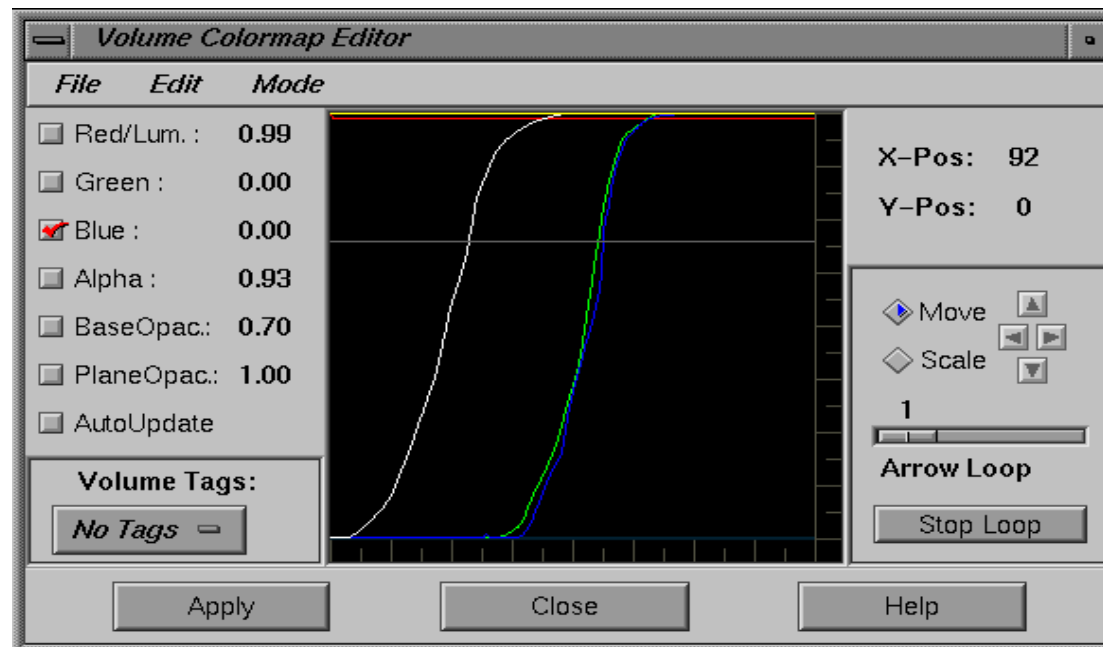
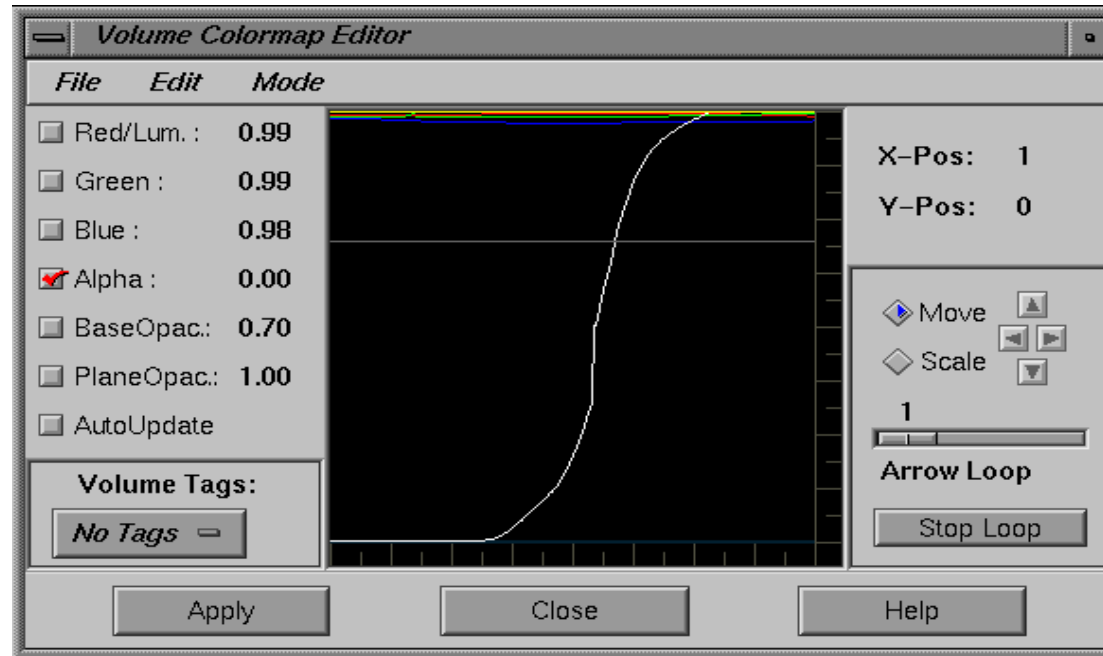


Classification: Transfer Function

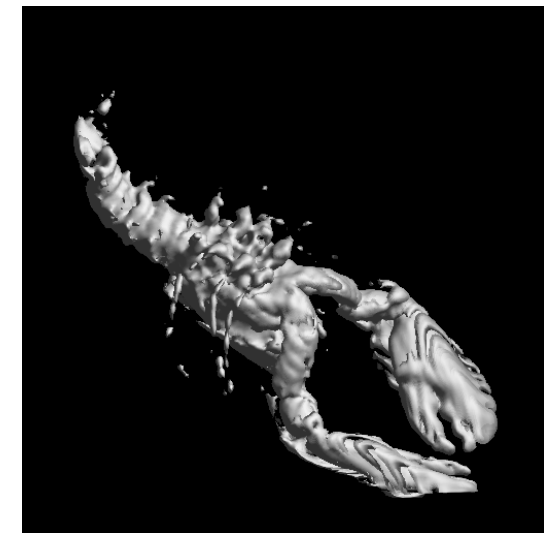
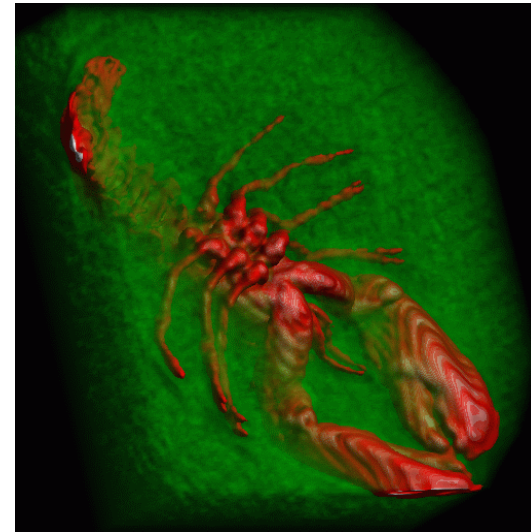
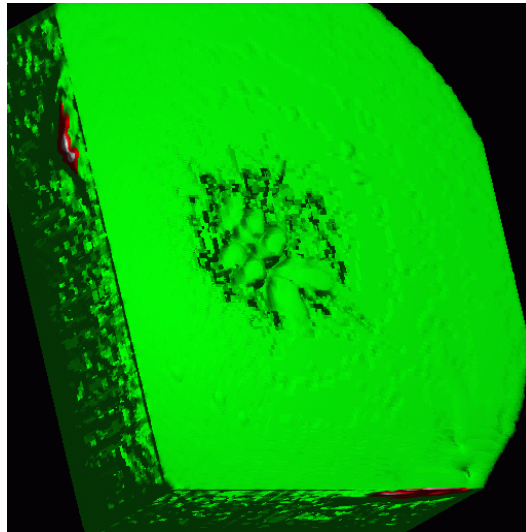
- Most widely used approach for transfer functions:
 - Assign to each scalar value a different color value
 - Assignment via transfer function T
 $T : \text{scalarvalue} \rightarrow \text{colorvalue}$
 - Common choice for color representation: RGBA
 - Code color values into a color lookup table



Classification: Example

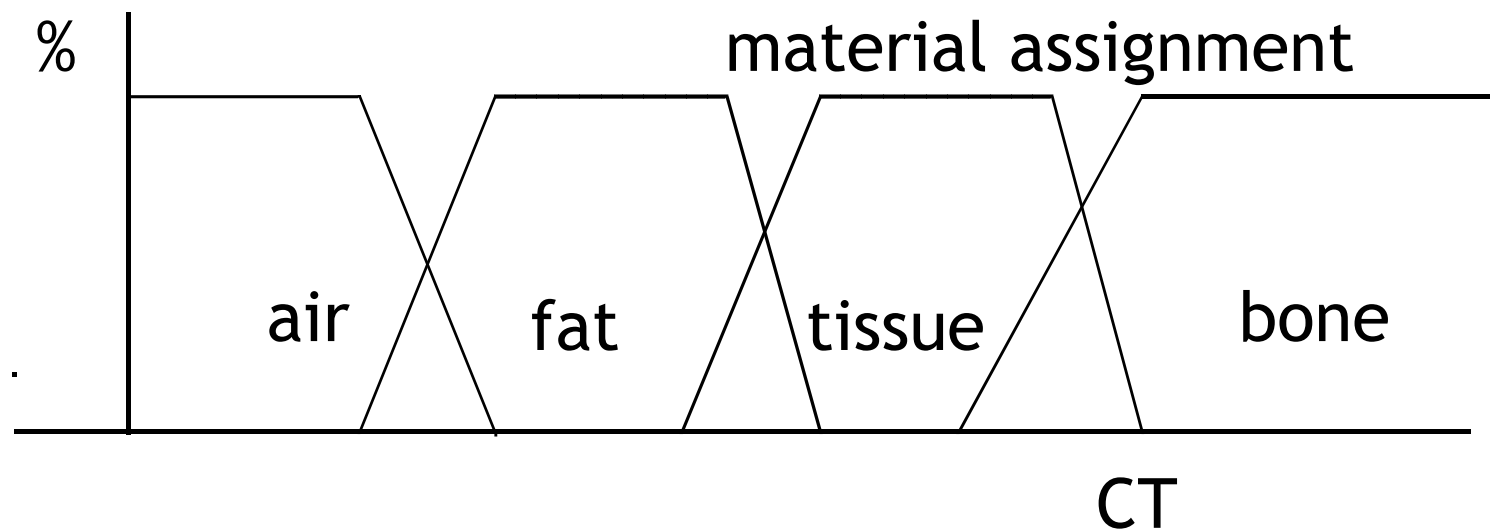
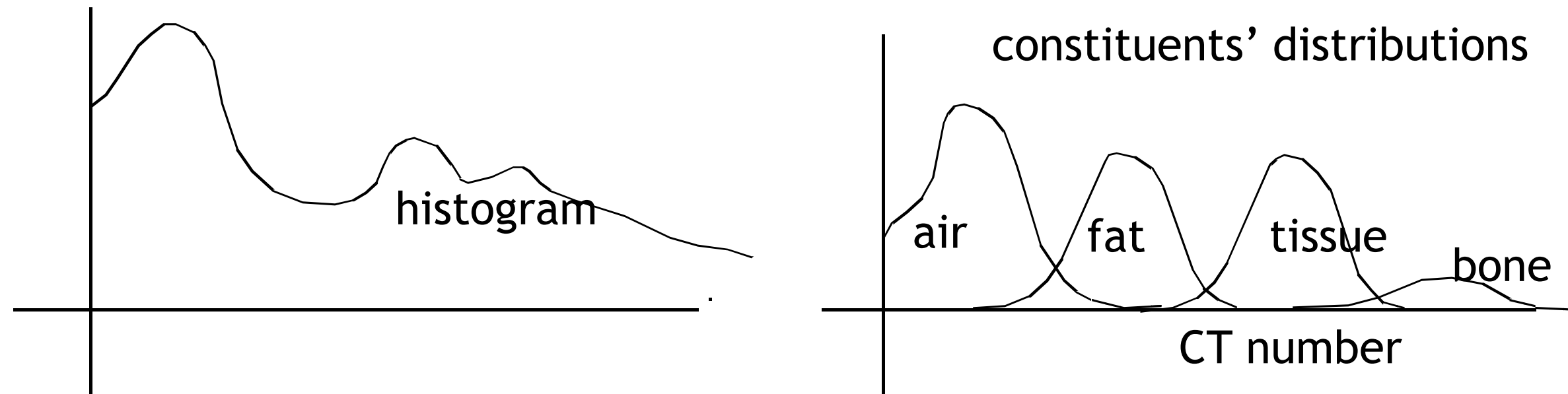


Classification: Example



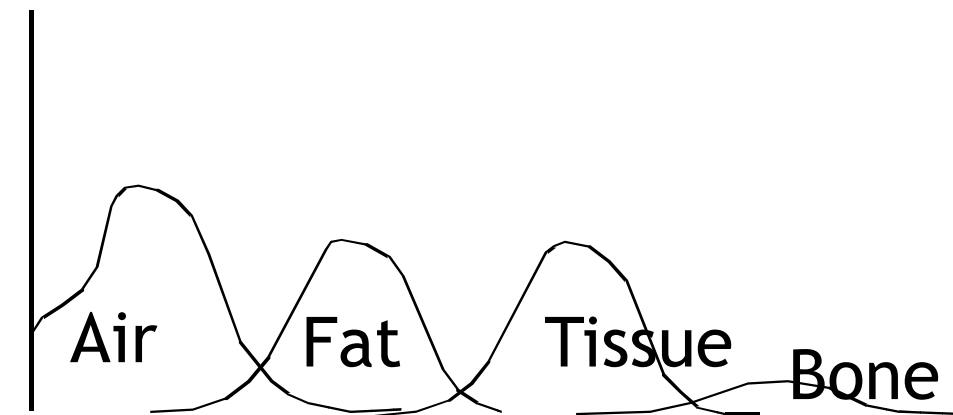
Classification in Medical Imaging

- Heuristic approach, based on measurements of many data sets



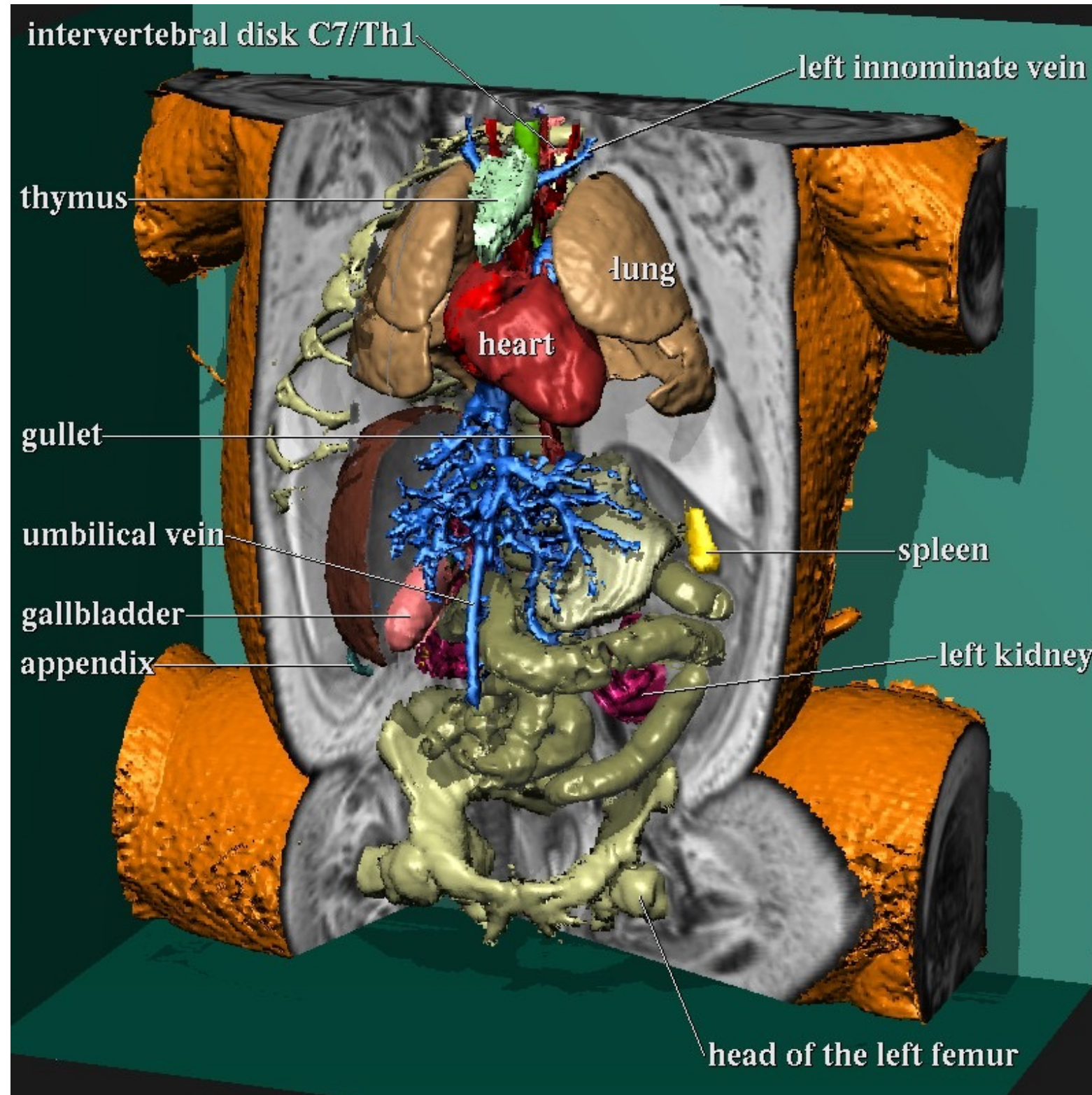
Problem: Segmentation

- Different features with same value
 - Example CT: different organs have similar X-ray absorption
 - Classification cannot be distinguished
- Label grid cells (or grid points) indicating a type
- Segmentation = pre-processing
- Semi-automatic process



→ Vast body of literature in medical imaging, image processing, etc.

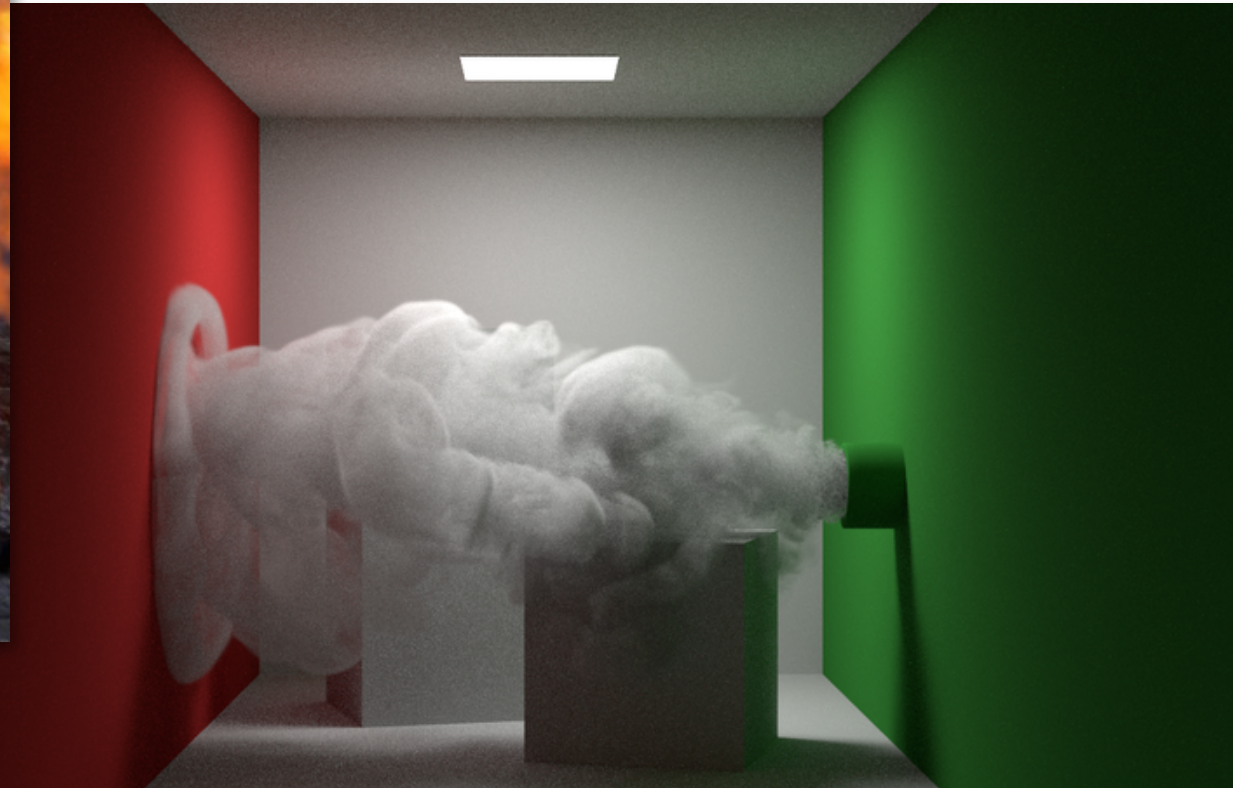
Segmentation



Anatomic atlas

Volume Rendering: Examples

- Smoke, fire, clouds, fluid effects etc.



Volume Rendering – Wrap-up

- Direct volume rendering can be implemented on the GPU
 - Ray traversal in fragment shader
 - Efficient scalar value retrieval using 3D textures
- Volume illumination
 - Use e.g. Phong illumination model or secondary rays
 - Often needs normals \rightarrow gradient of the scalar field



Image source: D. Jönsson et al., CGF 33(19), 2014.